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Research Books:

R [RB1] *"The Feynman Integral and Feynman's Operational Calculus",* Oxford Mathematical Monographs, Oxford Science Publications, *Oxford University Press,* Oxford, London and New York, approx. 800 pages (precisely, 771 + (xviii) pages & 21 illustrations), March 2000. ISBN 0 19 853574 0 (Hbk). (With Gerald W. Johnson.) [Corrected Reprinting, Jan. 2001. ***First Paperback Edition****, Jan. 2002*. ISBN 0 19 851572 3 (Pbk). Second Reprinting: Jan. 2003. Electronic Edition: forthcoming.] US Library of Congress Classification: QA312.J54 2000.

[This research treatise develops a mathematical theory of the beautiful but challenging subject of the Feynman path integral approach to quantum physics, and of the closely related topic of Feynman's operational calculus for noncommuting operators. It was written over a period of about ten years (Dec. 1989--Dec. 1999) and provides the most complete mathematical treatment of these subjects to date.

Some advantages of the approaches to the Feynman integral which are treated in detail in this book are the following: the existence of the Feynman integral is established for very general potentials in all four cases; under more restrictive but still broad conditions, three of these Feynman integrals agree with one another and with the unitary group from the usual approach to quantum dynamics; these same three Feynman integrals possess pleasant stability properties. The background material in mathematics and physics that motivates the study of the Feynman integral and Feynman's operational calculus is discussed, and detailed proofs are provided for the central results. The last chapter discusses topics in contemporary physics and mathematics (including knot theory and low-dimensional topology) where heuristic Feynman integrals have played a significant role.]

Table of Contents of the Book [RB1]:

1.3: Relationship with the Motivating Physical Theories: Background and Quantum-Mechanical Models. *Physical background. Highly singular potentials. Time-dependent potentials. Phenomenological models: complex and nonlocal potentials. Prerequisites, new material, and organization of the book.*

Chp. 2: The Physical Phenomenon of Brownian Motion. 2.1: A Brief Historical Sketch. 2.2: Einstein's Probabilistic Formula.

Chp. 3: Wiener Measure. 3.1: There is No Reasonable Translation Invariant Measure on Wiener Space. 3.2: Construction of Wiener measure. 3.3: Wiener's Integration Formula and Applications. *Finitely-based functions. Applications. Axiomatic description of the Wiener process.* 3.4: Nondifferentiability of Wiener Paths. *d-dimensional Wiener measure and Wiener process.* 3.5: Appendix: Converse Measurability Results. 3.6: Appendix: $B(X \times Y) = B(X) \otimes B(Y)$.

Chp. 4: Scaling in Wiener Space and the Analytic Feynman Integral. 4.1: Quadratic Variation of Wiener Paths. 4.2: Scale Change in Wiener Space. 4.3: Translation Pathologies. 4.4: Scale-Invariant Measurable Functions. 4.5: The Scalar-Valued Analytic Feynman Integral. 4.6: The Nonexistence of Feynman's "Measure". 4.7: Appendix: Some Useful Gaussian-Type Integrals. 4.8: Appendix: Proof of Formula (4.2.3a).

Chp. 5: Stochastic Processes and the Wiener Process. 5.1: Stochastic Processes and Probability Measures on Function Spaces. 5.2: The Kolmogorov Consistency Theorem. 5.3: Two Realizations of the Wiener Process.

Chp. 6: Quantum Dynamics and the Shrödinger Equation. 6.1: Hamiltonian Approach to Quantum Dynamics. 6.2: Transitions Amplitudes and Measurements. 6.3: The Heisemberg Uncertainty Principle. 6.4: Hamiltonian for a System of Particles.

Chp. 7: The Feynman Integral: Heuristic Ideas and Mathematical Difficulties. 7.1: Introduction. 7.2: Feynman's Formula. *Connections with classical mechanics: the method of stationary phase.* 7.3: Heuristic Derivation of the Schrödinger Equation. 7.4: Feynman's Approximation Formula. 7.5: Nelson's Approach via the Trotter Product Formula. *The Trotter product formula.* 7.6: The Approach via Analytic Continuation.

Chp. 8: Semigroups of Operators: An Informal Introduction.

Chp. 9: Linear Semigroups of Operators. 9.1: Infinitesimal Generator. *Integral equation. Evolution equation. Closed unbounded operators.* 9.2: Examples of Semigroups and Their Generators. *The translation semigroup. The heat semigroup. The Poisson semigroup.* 9.3: The Resolvent. 9.4: Generation Theorems. *The Hille-Yosida theorem. Dissipative operators and the Lumer-Phillips theorem.* 9.5: Uniformly Continuous and Weakly Continuous Semigroups. 9.6: Self-Adjoint Operators, Unitary Groups and Stone's Theorem. 9.7: Perturbation Theorems.

Chp. 10: Unbounded Self-Adjoint Operators and Quadratic Forms. 10.1: Spectral Theorem for Unbounded Self-Adjoint Operators. *Multiplication operators. Three useful forms of the spectral theorem.* 10.2: Applications of the Spectral Theorem. *The free Hamiltonian* H_0. *The heat semigroup and unitary group. Standard cores for the free Hamiltonian. Imaginary resolvents.* 10.3: Representation Theorems for Unbounded Quadratic Forms. *Basic definitions and properties. Representation theorems for quadratic forms. The form sum of operators.* 10.4: Conditions on the Potential V for H_0-Form Boundedness.

Chp. 11: Product Formulas with Applications to the Feynman Integral. 11.1: Trotter and Chernoff Product Formulas. *Product formulas for unitary groups.* 11.2: Feynman Integrals via the Trotter Product Formula. *Criteria for essential self-adjointness of positive operators. A brief outline of distribution theory. Kato's distributional inequality. Essential self-adjointness of the Hamiltonian* $H = H_0 + V$. *Conditions on the potential V for* H_0*-operator -boundedness. Feynman integral via the Trotter product formula for unitary groups.* 11.3: Product Formula for Imaginary Resolvents. *Hypotheses and statement of the main result. Proof of the product formula. Consequences, extensions and open problems.* 11.4: Application to the Modified Feynman Integral. *Modified Feynman integral and Schrödinger equation with singular potential. Extensions: Riemannian manifolds and magnetic vector potentials.* 11.5: Dominated Convergence Theorem for the Modified Feynman Integral. *Preliminaries. Perturbation of form sums of self-adjoint operators. Application to a general dominated convergence theorem for Feynman integrals.* 11.6: The Modified Feynman Integral for Complex Potentials. *Product formula for imaginary resolvents of normal operators. Application to dissipative quantum systems.* 11.7: Appendix: Extended Vitali's Theorem with Application to Unitary Groups. *Extension of Vitali's theorem for sequences of analytic functions. Analytic continuation and product formula for unitary groups.*

Chp. 12: The Feynman-Kac Formula. 12.1: The Feynman-Kac Formula, the Heat Equation and the Wiener Integral. 12.2: Proof of the Feynman-Kac Formula. *Bounded potentials. Monotone convergence theorems for forms and integrals. Unbounded potentials.* 12.3: Consequences.

Chp. 13: Analytic-in-Time or -Mass Operator-Valued Feynman Integrals.
13.1: Introduction. 13.2: The Analytic-in-Time Operator-Valued Feynman Integral. 13.3: Proof of Existence. 13.4: The Feynman Integrals Compared with One Another and with the Unitary Group. Application to Stability Theorems. 13.5: The Analytic-in-Mass Operator-Valued Feynman Integral. *Definition of the analytic-in-mass operator-valued Feynman integral. Nelson's results. Haugsby's results for time-dependent, complex-valued potentials. Further extensions via a product formula for semigroups.* 13.6: The

Analytic-in-Mass Modified Feynman Integral. *Existence of the analytic-in-mass modified Feynman integral. Product formula for resolvents: the case of imaginary mass. Comparison with other analytic-in-mass Feynman integrals. Highly singular central potentials—the attractive inverse-square potential.* 13.7: The Analytic-in-Time Operator-Valued Feynman Integral via Additive Functionals of Brownian Motion. *Introductory remarks. The parallel with Section 13.3. Generalized signed measures. The generalized Kato class. Capacity on* P^d. *Smooth measures. Positive continuous additive functionals of Brownian motion. The relationship between smooth measures and PCAFs. The analytic-in-time operator-valued Feynman integral exists for elements of* $S - GK_d$. *Examples.*

Chp. 14: Feynman's Operational Calculus for Noncommuting Operators: An Introduction. 14.1: Functions of Operators. 14.2: The Rules for Feynman's Operational Calculus. *Feynman's time-ordering convention. Feynman's heuristic rules. Two elementary examples.* 14.3: Time-Ordered Peturbation Series. *Perturbation series via Feynman's operational calculus. Perturbation series via a path integral. The origins of Feynman's operational calculus.* 14.4: Making Feynman's Operational Calculus Rigorous. *I: Rigor via path integrals. II: Well-defined and useful formulas arrived at via Feynman's heuristic rules. III: A general theory of Feynman's operational calculus with computations which are rigorous at every stage.* 14.5: Feynman's Operational Calculus via Wiener and Feynman Integrals: Comments on Chapters 15-18.

Chp. 15: Generalized Dyson Series, the Feynman Integral and Feynman's Operational Calculus. 15.1: Introduction. 15.2: The Analytic Operator-Valued Feynman Integral. *Notation and definitions. The analytic (in mass) operator-valued Feynman integral K. Preliminary results.* 15.3: A Simple Generalized Dyson Series ($\eta = \mu + \omega\, \delta_\tau\}$). *The Classical Dyson series.* 15.4: Generalized Dyson Series: The General Case. 15.5: Disentangling via Perturbation Expansions: Examples. *A single measure and potential. Several measures and potentials.* 15.6: Generalized Feynman Diagrams. 15.7: Commutative Banach Algebras of Functionals. *The disentangling algebras* A_t. *The time-reversal map on* A_t *and the natural physical ordering. Connections with Feynman's operational calculus.*

Chp. 16: Stability Results. 16.1: Stability in the Potentials. 16.2: Stability in the Measures.

Chp. 17: The Feynman-Kac Formula with a Lebesgue-Stieltjes Measure and Feynman's Operational Calculus. 17.1: Introduction. *Notation and hypotheses.* 17.2: The Feynman-Kac Formula with a Lebesgue-Stieltjes Measure: Finitely Supported Discrete Part ν. *Integral equation (integrated form of the evolution equation). Differential equation (differential form of the evolution equation). Discontinuities (in time) of the* *solution. Propagator and explicit solution.* 17.3: Derivation of the Integral Equation in a Simple Case ($\eta = \mu + \omega\, \delta_\tau$). *Sketch of the proof when v is finitely supported.* 17.4: Discontinuities of the Solution to the Evolution Equation. *The time discontinuities. Differential equation and change of initial condition.* 17.5: Explicit Solution and Physical Interpretations. *Continuous measure: uniqueness of the solution. Measure with finitely supported discrete part: propagator and explicit solution. Physical interpretations in the quantum-mechanical case. Physical interpretations in the diffusion case. Further connections with Feynman's operational calculus.* 17.6: The Feynman-Kac Formula with a Lebesgue-Stieltjes Measure: The General Case (Arbitrary Measure η). *Integral equation (integrated form of the evolution equation). Basic properties of the solution to the integral equation. Quantum-mechanical case: reformulation in the interaction (or Dirac) picture. Product integral representation of the solution. Distributional differential equation (true differential form of the evolution equation). Unitary propagators. Scattering matrix and improper product integral. Sketch of the proof of the integral equation.*

Chp. 18: Noncommutative Operations on Wiener Functionals, Disentangling Algebras and Feynman's Operational Calculus. 18.1: Introduction. 18.2: Preliminaries: Maps, Measures and Measurability. 18.3: The Noncommutative Operations * and +°. 18.4: The Functional Integrals K_λ^t and the Operations * and +°. 18.5: The Disentangling Algebras A_t, the Operations * and +°, and the Disentangling Process. *Examples: trigonometric, binomial and exponential formulas.* 18.6: Appendix: Quantization, Axiomatic Feynman's Operational Calculus, and Generalized Functional Integral. *Algebraic and analytic axioms. Consequences of the axioms. Examples: the disentangling algebras and analytic Feynman integrals.*

Chp. 19: Feynman's Operational Calculus and Evolution Equations. 19.1: Introduction and Hypotheses. *Feynman's operational calculus as a generalized path integral. Exponentials of sums of noncommuting operators. Disentangling exponentials of sums via perturbation series. Local and nonlocal potentials. Hypotheses.* 19.2: Disentangling exp[-tα + \int_{0}^{t} β(s) μ(ds)]. 19.3: Disentangling exp[-tα + \int_{0}^{t} β_1(s) μ_1(ds) + ∝ + \int_{0}^{t} β_n(s) μ_n(ds)]. 19.4: Convergence of the Disentangled Series. 19.5: The Evolution Equation. 19.6: Uniqueness of the Solution to the Evolution Equation. 19.7: Further Examples of the Disentangling Process. *Nonlocal potentials relevant to phenomenological nuclear theory.*

Chp. 20: Further Work on or Related to the Feynman Integral.
20.1: Transform Approaches to the Feynman Integral. References to Further Approaches.
A. The Fresnel Integral and Other Transform Approaches to the Feynman Integral. *The Fresnel integral. Properties of the Fresnel integral. An approach to the Feynman integral via the Fresnel integral. Advantages and disadvantages of Fresnel integral approaches to the Feynman integral. The Feynman map. The Poisson process and transforms. A "Fresnel integral" on classical Wiener space. The Banach algebras Σ and*

$\Phi(H_I)$ are the same. Consequences of the close relationship between Σ and $\Phi(H_I)$. More Functions in $\Phi(H_I)$. A unified theory of Fresnel integrals: introductory remarks. Background material. A unified theory of Fresnel integrals (continued). The Fresnel classes along with quadratic forms. The classes $\Gamma^q(H)$ and $\Gamma^q(B)$. Quadratic forms extended. Functions in the Fresnel class of an abstract Wiener space: examples of abstract Wiener spaces. Fourier-Feynman transforms, convolution, and the first variation for functions in Σ.

B. References to Further Approaches to the Feynman Integral.

20.2: The Influence of Heuristic Feynman Integrals on Contemporary Mathematics and Physics: Some Examples.

A. Knot Invariants and Low-Dimensional Topology.

The Jones polynomial invariant for knots and links. Witten's topological invariants via Feynman path integrals. Further developments: Vassiliev invariants and the Kontsevich integral.

B. Further Comments and References on Subjects Related to the Feynman Integral.

Supersymmetric Feynman path integrals and the Atiyah-Singer index theorem. Deformation quantization: star products and perturbation series. Gauge field theory and Feynman path integrals. String theory: Feynman-Polyakov integrals, and dualities. What lies ahead? Towards a geometrization of Feynman path integrals.

References.

Index of Symbols. Author Index. Subject Index.

R[RB2] *"Fractal Geometry and Number Theory"*. (Subtitle: *"Complex Dimensions of Fractal Strings and Zeros of Zeta Functions"*.) Research Monograph, Birkhäuser-Verlag, Boston, approx. 300 pages (precisely, 268 + (xii) pages & 26 illustrations), January 2000. (With Machiel van Frankenhuijsen.) ISBN 0-8176-4098-3. (Boston). ISBN 3-7643-4098-3 (Basel). US Library of Congress Classification: QA614.86.L36 1999.

[This (refereed) research monograph consists entirely of new research carried out by the authors over a period of about five years (March 1995--Nov. 1999). It develops a mathematical theory of 'complex dimensions' of fractal strings, and of the oscillations intrinsic to the geometry and the spectrum of the associated fractals. In particular, new results about the critical zeros of zeta functions are established and a geometric reformulation of the (Extended) Riemann Hypothesis is obtained in terms of the notion of complex dimension and of the frequency spectrum of fractal strings. On the fractal side, for example, precise explicit formulas are obtained for the volume of tubular neighborhoods of self-similar (and other fractal) strings.

In the long-term, this work is aimed at putting fractal geometry in an arithmetic context and, conversely, at putting various aspects of number theory (such as the theory of

Dirichlet series, for example) in a geometric framework.]

R[RB3] "*Fractal Geometry, Complex Dimensions and Zeta Functions*", (Subtitle: "*Geometry and Spectra of Fractal Strings"*.) Refereed Research Monograph, Springer Monographs in Mathematics, Springer-Verlag, New York, approx. 490 pages (precisely, 460 + (xxiv) pages & 54 illustrations), August 2006. (With Machiel van Frankenhuysen.)
ISBN-10: 0-387-33285-5. e-ISBN: 0-387-35208-2. ISBN-13: 978-0-387-33285-7.
US Library of Congress Control Number: 2006929212.

This is a sequel to and a greatly expanded version of the theory of complex fractal dimensions first developed in [RB2]. It contains a large amount of new research material (including several new chapters, sections, and appendices, along with many new examples and applications), almost all of which is directly connected to the work of the authors (and their collaborators) since the publication of [RB2], and a portion of which appeared in print for the first time in the present book.

Overview (of the approx. 200 pages of new material): New chapters on 'self-similar flows' (Chp. 7) and on 'quasiperiodic patterns of self-similar strings' (Chp. 3), providing a much more precise understanding (than in [RB2]) of the complex dimensions of (nonlattice) self-similar fractals (in $\mathbb{R}$) and dynamical systems, as well as of the error terms in the associated 'explicit formulas'. A new geometric description of self-similar fractal strings (in §2.1.1) and a discussion of self-similar strings with multiple generators (in Chp. 2 and throughout the rest of the book). New (and previously entirely unpublished) section (§6.3.3) on an (operator-valued) Euler product attached to the spectrum of a fractal string (generalizing that for the Riemann zeta function $\zeta(s)$ but also converging in the critical strip $0 < \text{Re}\, s < 1$); study of the eigenvalue spectrum of the corresponding 'spectral operator'. New pointwise tube formulas (§8.1.1), with a number of new applications to tube formulas for self-similar strings (in §8.4); new results on average Minkowski content (§8.4.3) and on error terms for nonlattice strings (§8.4.4). New sections (§11.1.1 and §11.4.1) on recent results on zeros of zeta functions in finite arithmetic progressions (extending those of the authors in §9.2 and §9.3 of Chapter 9 of [RB2]). Discussion in the last chapter (Chp. 12) of a number of new topics, including recent results on random fractal strings, quantized strings (fractal membranes), and especially on the complex dimensions of the Koch snowflake curve and its generalizations (via an associated 'tube formula'), as well as of an outline of a higher-dimensional theory of complex dimensions of self-similar systems and fractals. Also, further discussion of a possible cohomological interpretation of the complex dimensions. New appendix (App. A) on Nevanlinna theory and its applications in this context, and a new section (§B.4 of App. B) on 'two-variable zeta functions'. New examples, illustrations, theorems, and proofs, scattered throughout the book.]

Table of Contents of the Book [RB3]:

geometers' intuition of fractality. 12.2.2: Our definition of fractality. 12.2.3: Possible connections with the notion of lacunarity. 12.3: Fractality and Self-Similarity. *12.3.1: Complex dimensions and tube formula for the Koch snowflake curve. 12.3.2: Towards a higher-dimensional theory of complex dimensions.* 12.4: Random and Quantized Fractal Strings. *12.4.1: Random fractal strings and their zeta functions. 12.4.2: Fractal membranes: quantized fractal strings.* 12.5: The Spectrum of a Fractal Drum. *12.5.1: The Weyl-Berry conjecture. 12.5.2: The spectrum of a self-similar drum. 12.5.3: Spectrum and periodic orbits.* 12.6: The Complex Dimensions as the Spectrum of Shifts. 12.7: The Complex Dimensions as Geometric Invariants. *12.7.1: Connections with varieties over finite fields. 12.7.2: Complex cohomology of self-similar strings.* 12.8: Notes.

Appendix A: Zeta Functions in Number Theory. A.1. The Dedekind Zeta Function. A.2. Characters and Hecke L-Series. A.3. Completion of L-Series, Functional Equation. A.4. Epstein Zeta Functions. A.5. Two-Variable Zeta Functions. *A.5.1: The zeta function of Pellikaan. A.5.2: The zeta function of Schoof and van der Geer.* A.6: Other Zeta Functions in Number Theory.

Appendix B: Zeta Functions of Laplacians and Spectral Asymptotics. B.1: Weyl's Asymptotic Formula. B.2: Heat Asymptotic Expansion. B.3: The Spectral Zeta Function and Its Poles. B.4: Extensions. *B.4.1: Monotonic Second Term.* B.5: Notes.

Appendix C: An Application of Nevanlinna Theory. C.1: The Nevanlinna Height. C.2: Complex Zeros of Dirichlet Polynomials.

Bibliography. Acknowledgements.

Conventions. Index of Symbols. Author Index. Subject Index.

R [RB4] *"In Search of the Riemann Zeros: Strings, fractal membranes and noncommutative spacetimes",* Amer. Math. Soc., Providence, R I, 2008, 600 pages (precisely, 558+(xxix) pp.), February, 2008. ISBN-10: 0-8218-422-5. US Library of Congress Classification: QA333.L37 2007].

[The (physically motivated) theory proposed and developed in this research monograph represents approximately ten years of the author's research on this subject (between about 1996 and 2006). It realizes a synthesis of aspects of string theory, noncommutative geometry, the author (and his collaborators)' theory of fractal strings (now 'quantized' and referred to in this new form as 'fractal membranes') and their complex dimensions, as well as of number theory and arithmetic geometry. In particular, it builds upon and expands—but is also in many ways quite different from—the author's earlier (joint) work in [JA24-27] or in [RB2, RB3]. Much of the material presented in the main part of this book (with the exception of a portion of Chapter 2 and the first section of Chapter 3) is original and published for the first time.]

Table of Contents of the Book [RB4]:

generalizations. F.3.2: Translation dynamical system: the hull of a quasicrystal. F.3.3: Typical properties of atomic configurations. F.3.4: The noncommutative Brillouin zone (and groupoid C-algebra) of a quasicrystal.* F.4: Notes.

Acknowledgements. Bibliography.

Conventions. Index of Symbols. Author Index. Subject Index.

Refereed Edited Research Books:

[EB1] ***Editor:*** *"Progress in Inverse Spectral Geometry",* Proc. Summer School on "Progress in Inverse Spectral Geometry", held in Stockholm (Sweden) in June-July 1994, *Trends in Mathematics Series*, Vol. 1, Birkhäuser-Verlag, Basel, November 1997, 204 pages, (Co-Editor: Stig I. Andersson). ISBN 3-7643-5755-X (Basel). ISBN 0-8176-5755-X (Boston). US Library of Congress Classification: QA614.95.P78 1997.

[This volume gathers original research contributions or survey expository articles by some of the best experts in spectral geometry, along with a few papers by promising junior investigators selected by the editors. With two exceptions, the editors have reviewed and edited each paper individually.]

[EB2] ***Coordinating Editor:*** *"Harmonic Analysis and Nonlinear Differential Equations"*, (Volume in honor of Prof. Victor L. Shapiro), *Contemporary Mathematics,* Vol. 208, American Mathematical Society, Providence, RI, August 1997, 350 + (xii) pages. (Includes a Dedication and a Preface.) (Co-Editors: Lawrence H. Harper and Adolfo J. Rumbos.) ISBN 0-8218-0565-7 (alk. paper). US Library of Congress Classification: QA403.H223 1997.

[This volume gathers high quality original research contributions or survey expository articles by some of the leading experts in classical and modern analysis, working in the areas of harmonic analysis, nonlinear partial differential and mathematical physics. Each contribution in this volume has been individually refereed according to strict standards set by the American Mathematical Society.]

[EB3] ***Editor:*** *"Dynamical, Spectral and Arithmetic Zeta Functions",* Proc. Special Session (Amer. Math. Soc. Annual Meeting, San Antonio, Texas, Jan. 1999), *Contemporary Mathematics,* Vol. 290, American Mathematical Society, Providence, RI, December 2001, 195 + (x) pages. (Co-Editor: Machiel van Frankenhuysen.) ISBN 0-8218-2079-6 (alk. paper). US Library of Congress Classification: QA351.A73 1999.

[This volume gathers high quality original research contributions or survey expository articles by some of the leading experts working at the interface of number theory, dynamical systems and/or spectral as well as arithmetic geometry. Each contribution in the volume has been individually refereed according to strict standards set by the American Mathematical Society.]

[EB4] & [EB5]:

Managing Editor: "*Fractal Geometry and Applications*". (Subtitle: "*A Jubilee of Benoît Mandelbrot*".) [Two volumes (Parts 1 & 2), totaling approximately 1,100 pages (precisely, 1,080 + (xxvi) pages).] *Proceedings of Symposia in Pure Mathematics* (PSPUM), American Mathematical Society, Vol. 72, Parts 1 & 2, Providence, RI, Dec. 2004. (Co-Editor: Machiel van Frankenhuijsen.) ISBN 0-8218-3292-1 (set: acid free paper). ISBN 0-8218-3637-4 (part 1: acid free paper). ISBN 0-8218-3638-2 (part 2: acid free paper). US Library of Congress Classification: QA325.F73 2004.

Subtitle of Part 1: *Analysis, Number Theory, and Dynamical Systems*. (Approx. 520 pages; precisely, 508 + (xiii) pages.)

Subtitle of Part 2: *Multifractals, Probability and Statistical Mechanics, Applications*. (Approx. 560 pages; precisely, 546 + (xiii) pages.)

[The PSPUM Series is the most prestigious proceedings series published by the American Mathematical Society. Very few volumes are published every year (or decade). (The preparation of the present two-part volume has taken about three years.) In part for this reason, as the Managing Editor of the volume (i.e., of both Parts 1 and 2), I have devoted a great deal of attention and care to the selection of the invited contributors, the refereeing process (drawing upon more than forty expert referees), and the editing of the volume.. This has been an extremely time-consuming task for me, spanning over hundreds of hours, but one that I think will be ultimately worthwhile and useful to the mathematical as well as the broader scientific community.

The goal of these two books is to give an overview of the field of fractal geometry and of its applications [within mathematics (e.g., harmonic analysis, dynamical systems, number theory, probability, and mathematical physics) as well as to the other sciences (e.g., physics, chemistry, engineering, and computer graphics)], via a careful selection of research expository articles, tutorial articles, and original research papers. It should be accessible and useful to experts and non-experts alike.]

Books in Preparation:

[RB5] **Research Monograph:** *"Noncommutativity and Time-Ordering". (Subtitle:*

"Feynman's Operational Calculus and Beyond".) (With Gerald W. Johnson and Lance Nielsen.)

[TB] **Textbook:** *"An Invitation to Fractal Geometry and Its Applications"*. (With Dana Clahane, Erin Pearse and Robert Niemeyer.)

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