Research Books:


[This research treatise develops a mathematical theory of the beautiful but challenging subject of the Feynman path integral approach to quantum physics, and of the closely related topic of Feynman’s operational calculus for noncommuting operators. It was written over a period of about ten years (Dec. 1989–Dec. 1999) and provides the most complete mathematical treatment of these subjects to date.

Some advantages of the approaches to the Feynman integral which are treated in detail in this book are the following: the existence of the Feynman integral is established for very general potentials in all four cases; under more restrictive but still broad conditions, three of these Feynman integrals agree with one another and with the unitary group from the usual approach to quantum dynamics; these same three Feynman integrals possess pleasant stability properties. The background material in mathematics and physics that motivates the study of the Feynman integral and Feynman’s operational calculus is discussed, and detailed proofs are provided for the central results. The last chapter discusses topics in contemporary physics and mathematics (including knot theory and low-dimensional topology) where heuristic Feynman integrals have played a significant role.]

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Chp. 15: Generalized Dyson Series, the Feynman Integral and Feynman’s Operational Calculus.  15.1: Introduction.  15.2: The Analytic Operator-Valued Feynman Integral.  Notation and definitions.  The analytic (in mass) operator-valued Feynman integral $K$.  Preliminary results.  15.3: A Simple Generalized Dyson Series ($\eta = \mu + \omega \delta_\tau$).  The Classical Dyson series.  15.4: Generalized Dyson Series: The General Case.  15.5: Disentangling via Perturbation Expansions: Examples.  A single measure and potential.  Several measures and potentials.  15.6: Generalized Feynman Diagrams.  15.7: Commutative Banach Algebras of Functionals.  The disentangling algebras $A_\tau$.  The time-reversal map on $A_\tau$ and the natural physical ordering.  Connections with Feynman’s operational calculus.


solution. Propagator and explicit solution. 17.3: Derivation of the Integral Equation in a Simple Case \( (\eta = \mu + \omega \delta_t) \). Sketch of the proof when \( \nu \) is finitely supported. 17.4: Discontinuities of the Solution to the Evolution Equation. The time discontinuities. Differential equation and change of initial condition. 17.5: Explicit Solution and Physical Interpretations. Continuous measure: uniqueness of the solution. Measure with finitely supported discrete part: propagator and explicit solution. Physical interpretations in the quantum-mechanical case. Physical interpretations in the diffusion case. Further connections with Feynman’s operational calculus. 17.6: The Feynman-Kac Formula with a Lebesgue-Stieltjes Measure: The General Case (Arbitrary Measure \( \eta \)). Integral equation (integrated form of the evolution equation). Basic properties of the solution to the integral equation. Quantum-mechanical case: reformulation in the interaction (or Dirac) picture. Product integral representation of the solution. Distributional differential equation (true differential form of the evolution equation). Unitary propagators. Scattering matrix and improper product integral. Sketch of the proof of the integral equation.


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**B. References to Further Approaches to the Feynman Integral.**


A. Knot Invariants and Low-Dimensional Topology.

The Jones polynomial invariant for knots and links. Witten’s topological invariants via Feynman path integrals. Further developments: Vassiliev invariants and the Kontsevich integral.

B. Further Comments and References on Subjects Related to the Feynman Integral.


References.

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[This refereed research monograph consists entirely of new research carried out by the authors over a period of about five years (March 1995--Nov. 1999). It develops a mathematical theory of ‘complex dimensions’ of fractal strings, and of the oscillations intrinsic to the geometry and the spectrum of the associated fractals. In particular, new results about the critical zeros of zeta functions are established and a geometric reformulation of the (Extended) Riemann Hypothesis is obtained in terms of the notion of complex dimension and of the frequency spectrum of fractal strings. On the fractal side, for example, precise explicit formulas are obtained for the volume of tubular neighborhoods of self-similar (and other fractal) strings.

In the long-term, this work is aimed at putting fractal geometry in an arithmetic context and, conversely, at putting various aspects of number theory (such as the theory of...
Dirichlet series, for example) in a geometric framework.

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Bibliography. Acknowledgements.

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[The (physically motivated) theory proposed and developed in this research monograph represents approximately ten years of the author’s research on this subject (between about 1996 and 2006). It realizes a synthesis of aspects of string theory, noncommutative geometry, the author (and his collaborators)’ theory of fractal strings (now ‘quantized’ and referred to in this new form as ‘fractal membranes’) and their complex dimensions, as well as of number theory and arithmetic geometry. In particular, it builds upon and expands—but is also in many ways quite different from—the author’s earlier (joint) work in [JA24-27] or in [RB2, RB3]. Much of the material presented in the main part of this book (with
the exception of a portion of Chapter 2 and the first section of Chapter 3) is original and published for the first time.]

Table of Contents of the Book [RB4]:

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Chp. 5: Towards an ‘Arithmetic Site’: Moduli Spaces of Fractal Strings and Membranes. 5.1: The Set of Penrose Tilings: Quantum Space as a Quotient Space. 5.2: The Moduli Space of Fractal Strings: A Natural Receptacle for Zeta Functions. 5.3: The Moduli Space of Fractal Membranes: A Quantized Moduli Space of Fractal Strings. 5.4: Arithmetic Site, Frobenius Flow and the Riemann Hypothesis. 5.4.1: The moduli space of fractal strings and Deninger’s arithmetic site. 5.4.2: The moduli space of fractal membranes and
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Acknowledgements. Bibliography.

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Refereed Edited Research Books:


(This volume gathers original research contributions or survey expository articles by some of the best experts in spectral geometry, along with a few papers by promising junior investigators selected by the editors. With two exceptions, the editors have reviewed and edited each paper individually.)


(This volume gathers high quality original research contributions or survey expository articles by some of the leading experts in classical and modern analysis, working in the areas of harmonic analysis, nonlinear partial differential and mathematical physics. Each contribution in this volume has been individually refereed according to strict standards set by the American Mathematical Society.)

[This volume gathers high quality original research contributions or survey expository articles by some of the leading experts working at the interface of number theory, dynamical systems and/or spectral as well as arithmetic geometry. Each contribution in the volume has been individually refereed according to strict standards set by the American Mathematical Society.]

[EB4] & [EB5]:


Subtitle of Part 1: *Analysis, Number Theory, and Dynamical Systems*. (Approx. 520 pages; precisely, 508 + (xiii) pages.)

Subtitle of Part 2: *Multifractals, Probability and Statistical Mechanics, Applications*. (Approx. 560 pages; precisely, 546 + (xiii) pages.)

[The PSPUM Series is the most prestigious proceedings series published by the American Mathematical Society. Very few volumes are published every year (or decade). (The preparation of the present two-part volume has taken about three years.) In part for this reason, as the Managing Editor of the volume (i.e., of both Parts 1 and 2), I have devoted a great deal of attention and care to the selection of the invited contributors, the refereeing process (drawing upon more than forty expert referees), and the editing of the volume. This has been an extremely time-consuming task for me, spanning over hundreds of hours, but one that I think will be ultimately worthwhile and useful to the mathematical as well as the broader scientific community.

The goal of these two books is to give an overview of the field of fractal geometry and of its applications [within mathematics (e.g., harmonic analysis, dynamical systems, number theory, probability, and mathematical physics) as well as to the other sciences (e.g., physics, chemistry, engineering, and computer graphics)], via a careful selection of research expository articles, tutorial articles, and original research papers. It should be accessible and useful to experts and non-experts alike.]

**Books in Preparation:**

[RB5] **Research Monograph:** “Noncommutativity and Time-Ordering”. (Subtitle:
"Feynman's Operational Calculus and Beyond". (With Gerald W. Johnson and Lance Nielsen.)

[TB]  **Textbook:** “An Invitation to Fractal Geometry and Its Applications”. (With Dana Clahane, Erin Pearse and Robert Niemeyer.)
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