Sciama's principle and the dynamics of galaxies
I: Sciama's principle

COLIN ROURKE

This series of papers is devoted to a new approach to the dynamics of galaxies using a precise formulation of Mach's principle due to Sciama [11]. In subsequent papers we will use this principle to find accurate models for the ubiquitous rotation curves of galaxies and the predominant spiral structure without the need for "dark matter".

This first paper discusses Sciama's approach to Mach's principle and formulates it in the way that we need it in subsequent papers.

85A05; 85D99, 85A40, 83F05

1 Mach's principle

In any dynamical theory there are certain privileged frames of reference in which the laws of Newtonian physics hold to first order. These frames are variously called "inertial frames" or "rest frames". They are characterised by the lack of forces corellated with acceleration or rotation. In Newtonian physics there is a universal inertial frame referred to as "absolute space" and in general relativity there is an axiom the "equivalence principle" which provides these frames. Berkeley [1] and Mach [6] criticised Newton's assumption of absolute space. Berkeley suggested that the local rest frame could be defined by distant "fixed" stars. Mach's book [6, Ch II.VI (p 271 ff)] contains a devastating critique of Newton's assumptions and is well worth reading. It was extremely influential and Einstein acknowledged a debt to his ideas. Mach's basic point is that one should never assume anything that is not directly connected to observations of some kind and in particular the concept of the local inertial frame must be defined in terms of (theoretically) observable quantities. In a short while we will see some detail from Mach.

But before moving on we need to mention a basic property of inertial frames. They are only defined up to uniform linear motion. Given any inertial frame, a frame which is in uniform linear motion with respect to the given frame is also an inertial frame. Thus when we refer to "the" inertial frame at a point P what we mean is an equivalence class of frames. Two frames in the class being in mutual uniform linear motion with respect to each other. (For this reason, calling them “rest” frames is highly misleading and I shall not use this terminology again.)

Mach’s ideas have passed into general circulation as “Mach’s principle” which is usually summarised as stating that the local concept of inertial frame is correlated with the distribution and motion of all the matter in the universe. However there are many other ways of interpreting the principle and there is a huge literature on the subject. At its weakest, the principle is interpreted as merely stating that all phenomena must have their origin in some material source (see eg [12]), and it has even been interpeted as an assumption about the nature of the big bang (Tod [14]).

For our purposes, we need a statement which is more precise than these but not so wide ranging. We just need a local version which applies to rotation of inertial frames but we need it to be quantified precisely.

2 Sciama’s principle

The version that we need is close to the version in Sciama’s thesis [11]. Sciama makes a bold attempt to base a full theory of dynamics on Mach’s principle. His idea is that every particle Q of matter (of mass m_Q) in the universe contributes to the inertial frame at every other point P. The contribution is (a) proportional to the mass m_Q and (b) inversely proportional to the distance r_Q between P and Q. In other words the contribution is

$$m_Q\,\mathrm{IF}_Q/r_Q,$$

where IF_Q means the inertial frame at Q. The idea is that this should be summed over “all the matter in the universe”.

To make sense of this sum we need to make a number of assumptions. Firstly, in order to add up contributions, we need to work in a linear framework and the simplest way to do this is to work with a perturbation of flat (Minkowski) space, which is exactly what Sciama does. The underlying Minkowski space provides us with reference frames at each point and we can measure the motion of any frame with respect to this standard.

Working within a perturbation of Minkowski space limits us to weak fields but it suffices for most of this work. When we work near the massive centre of a galaxy, we can instead use a perturbation of any spherically-symmetric metric, eg the Schwarzschild metric, which allows us to consider stronger fields. But at some future stage, the theory needs to be established on a better footing without any presupposed background.

Secondly, in order for the summation to converge, the "universe" needs either to be finite or to be "quasi-finite" in the sense that only a finite part contributes to the sum. More detail on this point is given below.

Finally, we need to keep r_Q from getting too small or else the contribution of m_Q will be far too large. For this, we can either ignore masses which are close to P since the factor $1/r$ implies that the sum is dominated by distant matter, see the discussion below, or, if we are close to a significant and very massive body (eg the BH at the centre of a galaxy), we can normalise as explained below.

To formulate the principle quantitatively use the notation NM_P for the the non-uniform motion of the inertial frame at P and ditto Q, in other words its acceleration and/or rotation measured with respect to to the local reference frame then the inertial frame at P is given by the reference frame plus NM_P and the principle states that

Sciama's principle
$$\mathrm{NM}_P = K\sum_Q \frac{m_Q}{r_Q}(\mathrm{NM}_Q).$$

This statement is digested from Sciama's introduction and the precise formulation in terms of the field [11, Equation (1), page 37]. In order to distinguish this from Mach's principle, we shall refer to it as *Sciama's principle*. Here K is a normalising factor which will be dicussed futher below.

Notice that this principle is completely symmetric. The effect of Q's motion on the inertial frame (IF) at P is exactly similar to the effect of P's on the frame at Q. And note that the effect is *coherent* in the sense that an acceleration or rotation of the frame at Q causes an acceleration or rotation of the frame at P with the *same* direction or sense. Sciama describes this symmetry eloquently in his introduction, for example: *"... the statement that the Earth is rotating and the rest of the universe is at rest should lead to the same dynamical consequences as the statement that the universe is rotating and the Earth is at rest, ... "*

Also notice that using NM_P in the summation implies that we are ignoring inertial effect of matter in uniform linear motion. This is correct for small masses or for larger masses sufficiently distant that gravitational induction effects can be ignored.

With a caveat that this needs needs to be treated with care in special cases, we shall adopt this as a working hypothesis which fits the intuitive idea of inertial effects:

Working hypothesis *Uniform linear motion has no inertial effect.*

Sciama is clear that his principle is incompatible with general relativity (GR) and is attempting to create an alternative theory. Later we shall see precisely how the principle is incompatible with GR.

Sciama starts to derive a full gravitational theory from this principle. He specialises to a "field" (a vector field) defined on Minkowski space and as he makes clear this a very much an interim approach which will need improvement is a subsequent promised sequel paper. In order for the summation to converge, the "universe" needs either to be finite or to be "quasi-finite" in the sense that only a finite part contributes to the sum. More detail on this point is given in the next paragraph.

Sciama discusses three cases in detail:

(a) The effect of distant matter on the local IF.

The factor $1/r$ is chosen to make distant matter dominate. In order to get a finite sum, Sciama assumes standard Hubble expansion and then it is natural to limit the summation to the visible universe (in other words to ignore parts that are regressing faster than c). It is worth remarking in passing, that it is not necessary to assume the existence of a big bang (BB) to satisfy this quasi-finite hypothesis. There are models for the universe with redshift fitting observations but with no BB [4], the simplest of which is the expanding part of de Sitter space; there is also the (now largely ignored) continuous creation model of Hoyle et al [3]. The effect of distant matter needs to be normalised to unity. For example, if the whole universe is rotating about P with angular velocity ω, then we want this to induce a rotation of ω in the IF at P, in other words we want the situation to be exactly the same as if all were at rest. Similarly for acceleration. Thus we need

$$K \sum_Q \frac{m_Q}{r_Q} = 1 \tag{1}$$

where the sum is taken over all accessible matter Q (ie within the visible universe). One way to arrange this is to assume that $K = 1$ and

$$\sum_Q \frac{m_Q}{r_Q} = 1 \tag{2}$$

This makes perfect sense provided that we are not too close to any one mass (if we allow r_Q to tend to zero, the contribution from m_Q goes to infinity, which is absurd) and this is effectively what Sciama does. A more sensible way is to normalise by setting

$$K = 1/\sum_Q \frac{m_Q}{r_Q} \tag{3}$$

which compensates for large local masses and this is what we will do when we apply the principle near the large central mass of a galaxy.

Equation (2) implies a fundamental relation between the various gravitational and cosmological constants which Sciama derives as [11, Equation (7)]. He points out that this is, within reasonable limits, in accord with observations. Misner, Thorne and Wheeler (MTW) [7, below 21.160] make exactly the same point using more modern observations[1]. This provides a preliminary justification for the key factor $1/r$. A better justification comes with the simple model Sciama describes, where the field naturally decays like $1/r$. His model however is too simplistic (as he readily acknowledges) and in fact coincides with one of the standard approximations to GR, namely "gravitomagnetism". Shortly, we shall find other cogent reasons for the factor $1/r$.

For the other two cases he uses the model.

(b) A locally isolated mass. Here Sciama finds Newtonian attraction to first order (and in fact you always get attraction).

(c) A locally rotating frame. Here he finds the usual Newtonian story (Coriolis forces etc).

3 An excerpt from Mach's critique

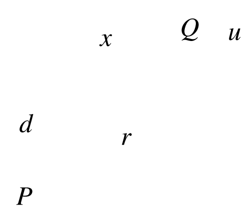

Figure 1: Proof of Mach's formula for apparent acceleration of bodies in uniform relative motion. Here $dx/dt = u$ (const) and $dr/dt = v$. From $r^2 = d^2 + x^2$ by differentiating twice we find $r\,d^2r/dt^2 + v^2 = u^2$ and hence $d^2r/dt^2 = (1/r)\,(u^2 - v^2)$ with $|v| = |(x/r)\,u| < |u|$.

There is a passage in Mach's critique which can be used to provide further support for the factor $1/r$. In Ch II.VI.7 (page 286) of [6] he points out that if two bodies move uniformly in ordinary 3–space then each sees the other as having non-zero acceleration along the common line of sight. Uniform motion does not *appear* uniform. Indeed if r is the distance between the bodies, then

$$\frac{d^2r}{dt^2} = \frac{1}{r}(u^2 - v^2) \tag{4}$$

[1]There are about 10^{11} galaxies in the visible universe of weight about 0.03 (10^{11} solar masses) at distances varying up to 10^{10}, where we use natural units ($G = c = 1$ and everything is in measured in years).

where u is the absolute value of the relative velocity and $v = dr/dt$. This is readily proved from Pythagoras' Theorem, see Figure 1. And notice that $|v| < |u|$. Thus to describe even uniform motion in terms of observation is quite complicated. On the next page he gives a formula for the mean acceleration of a body P with respect to a system of other masses (weighted by their mass) namely

$$\sum m_Q \frac{d^2 r_Q}{dt^2} \Big/ \sum m_Q \tag{5}$$

where m_Q is at distance r_Q from P. I have changed the notation (but not the formula) in order to show the connection of Mach's analysis with Sciama's principle. Because now, if we assume all bodies move uniformly with bounded mutual velocity and substitute (4) in (5), we obtain the formula for the acceleration of P in terms of the other masses

$$\sum \frac{m_Q}{r_Q} b_Q$$

where $b_Q = (u_Q^2 - v_Q^2)/\sum m_Q$ are all bounded by $1/\sum m_Q$ times the square of the bound for the mutual velocities. This is very close to Sciama's principle (ignoring rotation). To see the connection, drop the assumption of an absolute space where all this was supposed to take place. Keep only the observations. Then we can interpret this equation as specifying the "absolute" acceleration of P (and hence the IF at P) in terms of data at Q and if we were to label these data "inertial effect" then we would obtain precisely Sciama's principle.

It is important to remark that this discussion is not intended to suggest that uniform motion has an inertial effect; a small mass moving uniformly has negligible inertial effect, though a large mass has some effect due to inductive effects from its gravitational field. What is intended is that the formula that it is sensible to use to estimate the local inertial frame is likely to include a factor $1/r$ since apparent acceleration due to uniform motion does indeed include such a factor. The discussion is intended to support the contention that inertial effects drop off like $1/r$.

4 Rotation

From now on we shall concentrate on rotation of inertial frames. Sciama's principle applies equally to acceleration and rotation; thus rotation of a mass at Q contributes $K m_Q \omega_Q / r_Q$ to the rotation of the IF at P where ω_Q is the angular velocity of m_Q.

It is important to notice that it is the angular *velocity* of m_Q which contributes to the sum and not the angular *momentum* of m_Q about P. This behaviour (and a further final argument supporting the key factor $1/r$) can be deduced from a simple dimensional agument. There is a highly relevant passage in MTW [7] discussing precession of the Foucault pendulum which is worth quoting extensively. It starts on page 547 para 3 with the margin note *The dragging of the inertial frame*. I've edited it very slightly to make the notation fit with the present discussion and to suppress mention of conventional units. In this series of papers we always use *natural units*, with $G = c = 1$ and everything measured in years.

> *Enlarge the question. By the democratic principle that equal masses are created equal, the mass of the earth must come into the bookkeeping of the Foucault pendulum. Its plane of rotation must be dragged around with a slight angular velocity, ω_{drag}, relative to the so-called "fixed stars." How much is ω_{drag}? And how much would ω_{drag} be if the pendulum were surrounded by a rapidly spinning spherical shell of mass m_{shell} and radius r_{shell} turning at angular velocity ω_{shell}? Einstein's theory says that inertia is a manifestation of the geometry of spacetime. It also says that geometry is affected by the presence of matter to an extent proportional to the factor G/c^2* (ie 1 in natural units). *Simple dimensional considerations leave no room except to say that the rate of drag is proportional to an expression of the form*
>
> $$\omega_{\text{drag}} = k\frac{m_{\text{shell}}}{r_{\text{shell}}}\omega_{\text{shell}}. \tag{21.155}$$
>
> *Here k is a factor to be found only by detailed calculation. . . .*

Details of the dimensional argument used here will be given later. The authors continue by discussing the results of Lense and Thirring where k is calculated to be $4/3$ assuming a specific approximation which is in fact identical to the Sciama field. We shall have more to say about this shortly.

At this point it is worth making an observation. The Sciama field can be seen as a first approximation to a full-blown theory of dynamics based on Mach's principle. Since it coincides with gravitomagnetism, which is a first approximation to General Relativity, it follows that no local observations, where the fields are weak (for example the Gravity Probe B experiment [2]) can distinguish between General Relativity and a theory of dynamics based on Mach's principle. The main thrust of this set of papers is that there is however strong experimental evidence in favour of the latter from observations of galaxies.

5 The weak Sciama principle

We continue by analysing the MTW quotation and equation (21.155). Their "democratic principle" is close to Sciama's principle, at least in its universality, referring as it does to all (accessible) matter in the universe. The equation itself is precisely the principle for the contribution of the mass m_{shell}. And notice that it is implied that the dragging effect of the earth should be coherent with the earth's rotation. This point is so obvious that it may easily be overlooked and I only mention it because shortly we shall examine a model where the dragging is not always coherent. To see the connection with Sciama's principle for many distinct rotating masses, let's do some thought experiments. Replace the shell by a ring of matter at distance $r = r_{\text{shell}}$. Nothing changes qualitatively. The constant k reflects the precise geometry of the setup and may change. Now imagine that the ring is a necklace of n beads all of the same mass m. By the democratic principle, each has the same effect $\omega'_{\text{drag}} = \omega_{\text{drag}}/n$ and if P is the centre of the ring and Q one of the beads we now find that Q contributes $km\omega/r$ to the inertial frame at P where ω is the angular velocity of Q moving around P. But the local motion of Q is exactly the same as a (uniform) linear motion of velocity ωr along the tangent together with a rotation on the spot of ω. Using our working hypothesis, the linear motion has no inertial effect and we find exactly Sciama's principle in this case, namely:

Weak Sciama Principle *A mass m at distance r from P rotating with angular velocity ω contributes a rotation of $km\omega/r$ to the inertial frame at P where k is constant.*

We have arrived at the precise statement of Sciama's principle that we need for the dynamics of galaxies and we shall call this the *weak Sciama principle*. The constant k is a normalising factor which needs to be set in context. When we use the principle in the next paper [9, Section 2], we will make this precise.

Incidentally we can now see clearly why our working hypothesis implies that angular momentum is the wrong measure of the inertial effect of one mass on another. We are assuming that uniform linear motion has no inertial effect, but, adding a linear motion to Q may well have a strong effect on its angular momentum about P. Conversely rotation need not correlate with angular momentum: If Q is in fact a point mass, then rotation of Q with angular velocity ω has no angular momentum about P whereas motion in a circle around P with the same angular velocity does have angular momentum.

The weak Sciama principle is not Machian in even the weakest version (that all effects are due to observable source). It makes no attempt to completely specify the IF at P in terms of all the matter in the universe and indeed it leaves open the possibility that the IF at P may be affected by events that we may not know fully about (perhaps they are outside our visible horizon — cf MacKay–Rourke [5]). But the advantage of a local statement of this type is that it avoids the causality problems implicit in any global statement and it is open to direct verification using local observations. The main thesis of this series of papers is that it is indeed strongly supported by observations of galaxies and in particular their characteristic rotation curves.

6 The Lense–Thirring effect

Like the full Sciama principle, the weak principle only makes sense in an approximation to Minkowski space and this is exactly how we shall use it (the formulation of this is given at the start of part II). Here we continue by examining one of the attempts in the literature to formulate the dragging effect on inertial frames (which from now on we shall call *inertial drag*) due to a heavy rotating body, namely the early work of Lense and Thirring [13] mentioned above, who calculated the inertial drag due to a rotating spherical shell for points nearby. As we saw above, this effect is roughly in accord with Sciama's principle for points inside the shell, but as we shall see shortly, it is hopelessly wrong outside.

The approximation used here is the same as that used by Sciama and is known as gravitomagnetism. The equations correspond formally to Maxwell's equations and we can understand the effect by thinking of electromagnetism. Motion of matter corresponds to electrical current and a cicular motion induces a linear magnetic effect. The dragging effect corresponds to magnetic lines of force with the induced rotation having the line as axis with rotation around the line in the positive sense. Thus a rotating body behaves like a magnet and causes inertial drag which is coherent near the poles but anti-coherent to the side where the magnetic lines run back between the poles.

This has some very counter-intuitive consequences.

(a) Uniform linear motion has rotational inertial effects.

(b) A rotating body drags some frames nearby in the opposite direction to the rotation causing the drag.

(c) In general the direction of drag is unrelated to the rotation which induces it.

Effect (b) was picked up by Rindler [8] and correctly labelled "anti-Machian". However his conclusion that Mach's principle needs to be treated with care "one simply cannot trust Mach!" is bizarre. The philosophical reasons for Mach's principle are compelling and it must be incorporated in any theory that describes reality. It is the Lense–Thirring effect that must be wrong. In any case, it is not necessary to appeal to Mach's principle to see that inertial drag should be coherent. As we shall see in a couple of lines, a simple thought experiment using general principles of symmetry and continuity will establish this fact.

7 Central rotation

Perhaps the Lense–Thirring effect is wrong because of the approximation used, so now let's turn now to theories without approximation, including GR. Assume that we have a dynamical theory, which may not be GR, but which is metrically based and which specialises to special relativity locally in same way that GR does, with a similar equivalence principle. Here is a simple thought experiment which shows that, in any such theory, frame dragging due to a central rotating body exists and is coherent.

Imagine that the universe is a 3–sphere (spatially) and that it is filled with two very heavy bodies (both 3–balls) with a comparatively small gap between them. Suppose that these bodies are in relative rotation. Then by symmetry frames half way between the bodies will rotate at the average speed and by continuity the inertial drag will move towards rotation with each of the bodies as you move away from the centre. Diagramatically we have the situation pictured in Figure 2. Note that in the figure we have represented the bodies as nested. To get the correct view think of the outer circle labelled "infinity" as the diametrically opposite point to the centre of the inner body. To make sense of inertial drag here, we assume that the space between the two bodies has a flat background metric, but we do not assume anything about the space inside the bodies.

"infinity"

inner body

outer body

Figure 2: Inertial drag between two heavy bodies

Now shrink the inner body to be the central rotating body and imagine the outer body to be the rest of the heavy universe. It is unreasonable to suppose that the qualitative description of inertial drag changes during the shrinking process and therefore, in the final metric, the central body will induce coherent inertial drag.

We can now appeal to the same dimensional considerations as used in the passage quoted from MTW (above) to deduce the weak Sciama principle. Let the rotating body be labelled Q and have mass m and angular velocity ω. For simplicity, consider a point P on the equatorial plane of the rotating body at distance r from the centre. It is a commonsense assumption that the dragging effect at P is proportional to $m\omega$ and, being a pure rotation of the local inertial frame, has dimension $1/T$ where T means "time". (Notice that there is no sensible meaning to the centre of rotation for this effect. Two rotations which have the same angular velocity but different centres differ by a uniform linear motion and inertial frames are only defined up to uniform linear motion.) Now in relativity time, mass and distance all have the same dimension (this is true in special relativity and the dynamical theories that we are discussing here are all based on special relativity). Thus $m\omega$ is dimensionless and the only sensible formula for the induced inertial drag is $km\omega/r$, possibly normalised (note in passing that normalising constants such as K above are dimensionless and do not affect this argument).

We can go further with this thought experiment. Assume now that the universe is $\mathbb{R}^3$ spatially with our heavy inner rotating body at the origin. And imagine that the outer body is the outside of a sphere of radius R say and is in fact at rest. Now let R tend to infinity and, as it does so, control the mass of the outer body to keep its inertial effect near the inner body constant. In the limit, the outer body is replaced by an asymptotically flat metric near infinity and then, outside the inner body, we have a metric which is stationary (the whole construction was stationary) axially-symmetric and asymptotically flat at infinity. Assume now that the theory we are considering is in fact GR. In GR this metric must coincide with the Kerr metric which is well-known to be the unique metric with these properties. Equally well-known, the inertial drag effects of the Kerr metric drop off like $1/r^3$. Thus this metric, which we have constructed using the thought experiment, continuity and dimensional arguments, is not GR.

One may wonder why the dimensional argument does not equally apply to the Kerr metric. This is because the Kerr metric has a well-defined angular momentum but no well-defined angular velocity. Thus the inertial drag effect of the Kerr metric must be proportional to angular momentum NOT to mass times angular velocity. But angular momentum has dimension T^2 and to get a drag effect of dimension $1/T$ we need a formula of the type kA/r^3 where A is angular momentum.

8 The failure of GR

We have just seen that GR is not a good model of reality. It fails to provide a model for the neighbourhood of a rotating body in accord with the commonsense arguments used above. As this is a highly contentious statement, it's worth recapping the arguments here.

(1) The thought experiment of the nested heavy bodies (using MTW democracy principle) proves inertial drag exists and is coherent.

(2) The working hypothesis that linear uniform motion has no rotational inertial effect implies that inertial effects are proportional to mass times angular velocity.

(3) The simple dimensional argument of MTW proves that inertial drag drops off proportionately with $1/r$.

(4) The continuity construction pushing the outer body to infinity constructs a metric which is stationary, axisymmetric and flat at infinity with inertial drag dropping off like $1/r$.

But if the theory we are considering is GR then we must now have the Kerr metric with inertial drag dropping off proportionately with $1/r^3$ and we have a contradiction.

Of course there is nothing inconsistent in GR and the Kerr metric. It just that they contradict these commonsense considerations, which could indeed be misleading. In the end the acid test is experimental evidence. Does inertial drag in fact drop off like $1/r$ or $1/r^3$? Well the main thrust of this work is that there is indeed very strong experimental evidence for $1/r$ and this will be presented in the subsequent papers in the series.

9 Correcting GR

Before moving on to the next topic (in the next paper in the series) namely modelling the rotation curve of a galaxy, we pause very briefly to consider how GR might be ammended to make it model the neighbourhood of a rotating body with inertial drag dropping off like $1/r$. Obviously the metric approach of GR with space-times having metrics of signature $(-,+,+,+)$ is correct and equally obviously we must have the same equivalence principle based on special relativity. It is the actual equations of GR that need correcting. As currently formulated, GR deals well with acceleration and gravitational effects. It fails as we have seen to deal well with rotational effects. What is needed is for rotation to be directly coupled to the "field" just as stress and energy are coupled. As presently formulated, rotation couples to the field through motion and not through angular velocity. The corrected equations should reduce to the usual ones when there are is no (or only weak) rotation effects: for example the effects corresponding to say the rotation of the sun are very weak as we shall see in [9, Section 9]. The key effects for this work are due to the rotation of truly enormous masses at the centres of galaxies.

The basic equation for GR states that $G = T$ where G is the Einstein tensor (closely related to the Ricci curvature tensor) and T is the stress-energy tensor. This implies that a vacuum space-time is Ricci flat. What is needed is that a vacuum space-time can carry the curvature caused by inertial drag and hence not necessarily be Ricci flat. We need to ammend G so that it carries curvature due to inertial drag effects dropping of at $1/r$ and we need to ammend T to include rotational stress. Thus if there is no rotation and no inertial drag effects, the equations would be unaltered.[2] Some idea of the type of vacuum metric that ought to be allowable is given by the "interim metric" based on the Schwarzschild metric, described in [9, Section 4]. I hope to return to this topic in future work. For the purposes of this series of papers it is sufficient to work with a perturbation of flat Minkowski space which includes inertial drag effects and I will take this up at the beginning of the next paper.

10 Coda

Sciama's initiative, to base a dynamical theory on Mach's principle as formulated in what we have called "Sciama's principle", has never been followed up and this approach to dynamics remains dormant. One of the aims of this paper is to reawaken this approach. Sciama did return to the topic of Mach's principle in [12]. However this paper abandons Sciama's principle and formulates Mach's principle in one of its weakest forms, namely that all phenomena have their origin in some material source or boundary condition. Moreover the theory exposited in [12] is GR which as we have seen in incompatible with even the weak Sciama principle.

References

[1] **G Berkeley**, *The principles of human understanding* (1710), *De Motu* (1726)

[2] There is some experimental evidence that the Schwarzschild metric is correct when there is no rotation, given by the study [10] of certain quasars where a model based on the Schwarzschild metric yields numerical data in accord with observations.

[2] **C W F Everitt et al**, *Gravity Probe B: Final Results of a Space Experiment to Test General Relativity*, Phys. Rev. Lett. 106, 221101

[3] **F Hoyle**, **G Burbidge**, **J V Narlikar**, *A quasi-steady state cosmological model with creation of matter*, Astrophys J 410 (1993) 437–457 DOI:10.1086/172761

[4] **R S MacKay**, **C Rourke**, *Natural observer fields and redshift*, J. Cosmology 15 (2011) 6079–6099
http://msp.warwick.ac.uk/~cpr/paradigm/redshift-nat-final.pdf

[5] **R S MacKay**, **C Rourke**, *Gamma ray bursts*, submitted for publication, available at http://msp.warwick.ac.uk/~cpr/paradigm/GammaRayBursts.pdf

[6] **E Mach**, *The science of mechanics: a critical and historical account of its development*, Translation from the original German by T J McCormack, Open Court Publishing Co (1893) (page numbers in the text refer to the sixth American edition (1960) Lib. Congress cat. no. 60-10179)

[7] **C W Misner**, **K S Thorne**, **J A Wheeler**, *Gravitation*, Freeman (1973)

[8] **W Rindler**, *The Lense–Thirring effect exposed as anti-Machian*, Phys Lett A 187 (1994) 236–238

[9] **C Rourke**, *Sciama's principle and the dynamics of galaxies, II: The rotation curve*, submitted for publication, available at
http://msp.warwick.ac.uk/~cpr/paradigm/Sciama2.pdf

[10] **C Rourke**, **R Toala Enriques**, **R S MacKay**, *Black holes, redshift and quasars*, preprint, available at http://msp.warwick.ac.uk/~cpr/paradigm/quasars.pdf

[11] **D Sciama**, *On the origin of inertia*, Mon. Not. Roy. Astrom. Soc. 113 (1953) 34–42

[12] **D Sciama**, **P C Waylen**, **R C Gilman**, *Generally covariant integral formulation of Einstein's field equations*, Phys. Rev. 187 (1969) 1762–1766

[13] **H Thirring**, **J Lense**, *Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einstienschen Gravitationstheorie*, Phys Z. 19 (1918) 156–163

[14] **K P Tod**, *Mach's Principle revisited*, Gen. Rel. Grav. 26 (1994) 103–111

Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK

cpr@msp.warwick.ac.uk

http://msp.warwick.ac.uk/~cpr

Analytical Dynamics - by Firdaus E. Udwadia February 1996.Â There, we simply stated the principle as a basic principle of mechanics. We obtained the fundamental equation which described the motion of systems constrained by holonomic and nonholonomic constraints using this principle. It behooves us to understand Gauss's principle in greater depth now, moving full circle, as it were, by coming around to where we started from. This will be our agenda for this chapter.Â Using this principle, we will then provide an alternative proof for the fundamental equation in generalized coordinates. Statement of Gauss's Principle in Cartesian Coordinates. Consider a system of n particles. The Universal Principles of Educative instruction are chiefly realized through the contents of the texts and illustrations given in the text-books. The Principle of Consciousness. implies such process of teaching when the linguistic phenomena of the language are taught consciously by means of comparing, explanations, demonstrations, historical comments to make the process of acquision easier, to prevent interferenceÂ The principle of Activity (Activeness) requires activity of both, on the pant of the T and the learners: choral, individual work, work in small groups (problem - solving, project work, etc). But the T should bear in mind that students are different: extraverts and introverts display their activity differently. Sciama referred to the effect of the acceleration dependent term as Inertial Induction. In 1984 Amitabha Ghosh proposed adding a velocity dependent term to the static Newtonian term and Sciama's acceleration dependent term.[10] The name he applied to the effect was Velocity Dependent Inertial Induction. Variations in the statement of the principle[edit].Â Mach0: The universe, as represented by the average motion of distant galaxies, does not appear to rotate relative to local inertial frames. Mach1: Newton's gravitational constant G is a dynamical field. Mach2: An isolated body in otherwise empty space has no inertia.Â This textbook, among other writings by Sciama, helped revive interest in Mach's principle. Raine, D. J. (1975). "Mach's Principle in general relativity".