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The Many-Body Theory; its logic along the years
(Dedicated to the memory of O. Dumitrescu)

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Abstract

A brief history of the many-body theory is sketched, the non-integrability of the interacting particles is shown, many-particle forces are derived, and their status and relevance are assessed.

The beginning. The many-body theory is a non-relativistic quantum theory of ensembles of identical interacting particles. It appeared at the same time with the birth of Quantum Mechanics, probably in the fundamental papers by Born, Heisenberg, Jordan in 1925 and Schrodinger in 1926. The many-particle wavefunction $\psi(x_1, x_2, ...)$ has been introduced in those times as the fundamental means of describing interacting quantum particles. It is easier perhaps to think about one-particle wavefunction $\psi(x)$, but it seems more difficult to accept a many-body one. The entanglement involved there produces awe even today.

The symmetry of the Schrodinger equation with respect to particle permutations requires the many-body wavefunction $\psi(x_1, x_2, ...)$ be a representation of the permutation group, according to the spin; either totally symmetric for bosons or totally antisymmetric for fermions. Pauli's exclusion principle was incorporated in the many-body theory since its very beginning.

The Hartree-Fock. A great advance was made in the many-body theory at the beginning of the 30s with the Hartree-Fock equations. The underlying mean field was seen long ago by Bohr and employed extensively by Heitler for atoms. The exchange contribution for fermions, due to Fock, was a bit mysterious at the beginning, with its purely quantum origin. It allowed Heisenberg to give an earlier explanation to the magnetism. Dirac was so enthusiastic about the Hartree-Fock equations (and dealt much with the exchange), that he announced in the thirties that we had finally the equations for the matter, though complicated; it only remained to be solved.

Solving them extended over a substantial range of years, and runs even now with the nanostructures. It started with Pople around 1940 for molecules and continued with Kohn and Sham since 1960. Little progress has beeen recorded, except for a wide use of the electronic computers.

Unitentionally, the Hartree-Fock equations pointed to a much deeper aspect of the many-body theory: the one-particle elementary excitations. These have been understood much later, by Landau for instance in the 40-50s, and not even now by many others.

The many-body. In the fifties there was a "vigorous" revigoration of the field, and the name "many-body" has then been coined. In those years the famous RPA (random phase approximation) has been introduced by Bohm and Pines. The basic assumption in this approach is the treatment of the interaction by perturbation techniques in an otherwise one-particle picture. People capitalized in this endeavour on the field theory matured in those years, on Feynman diagrams, propagators and others alike, in the hope that these new powerful techniques would give new insights into, and solutions to the interacting particles. Meantime, superconductivity, superfluidity, the electron liquid, the normal Fermi liquid and many others received already their solution, without any many-body theory. In the 50s and later on a great deal of work was devoted to the interacting nucleons in the atomic nucleus, which caught even Bethe's attention (after he rationalized the theory of the atomic nucleus in 1936-37). The finite-temperature many-body formalism was rapidly set up, as well as the finite-temperature Green functions and the corresponding diagrammatic techniques. The famous book by Abrikosov, Gorkov, Dzyaloshinski which appeared in 1960 was the result of trying to understand more and deeper, by means of the new techniques, the previously solved many-body problems. There was not much progress with this understanding. The field has reached maturation and stagnation, as recorded by the classic book of Fetter and Walecka which appeared around 1970.

When it was my turn to enter Physics around 1970, I myself fell a prey to this decline. Soon, the many-body people with their many-body tools were moving to the one-dimensional physics, supported by the newly discovered quasi-one-dimensional materials (like the nuclearists go over today to nanostructures). Nevertheless, the many-body theory was a very convenient means to set up research programs, because it was able to accommodate generously both solid-state, condensed matter and nuclear physicists. And with a little stretch to a "few-body systems", we were able to include the particle physics too in such programs. A current joke at that time was "The One, Few and Many-Particle Theory". The nanostructures of today should be able to give a new motivation to this field (but it seems that it doesnt).

Non-integrability. Of course, the many-body theory originates in classical physics, in problems like celestial mechanics and the kinetic theory of gases. Poincare was probably the first who, around 1900, pointed out certain uncomfortable peculiarities in problems like the three-body motion of the Earth, the Sun and the Moon. The Statistical Mechanics of nowadays still struggles with the classical motion of interacting particles.

Let an ensemble of identical particles of mass m be described by a hamiltonian

$$H = \sum_i mv_i^2/2 + (1/2)\sum_{ij}{}' V_{ij} \tag{1}$$

written with velocities $\mathbf{v}_i$ and interaction V_{ij}, where i, j are labels for coordinates x_i, x_j, spin included. The pair-wise interaction V_{ij} is assumed to depend only on the distance $|\mathbf{x}_i - \mathbf{x}_j|$ and, occasionally, on spins. The prime on the sum means $i \neq j$. The equations of motion read

$$md\mathbf{v}_i/dt = -\sum_j{}' \partial V_{ij}/\partial \mathbf{x}_i \ . \tag{2}$$

It is easy to check that the total momentum $\sum m\mathbf{v}_i$ is conserved, as a consequence of the invariance under spatial translations. Similarly, the total energy H is conserved, as a consequence of the invariance under time translations. Likewise, the total angular momentum $L = \sum \mathbf{x}_i \times m\mathbf{v}_i$ is conserved, due to the invariance under spatial rotations.

Making use of $dV_{ij}/dt = (\partial V_{ij}/\partial \mathbf{x}_i)\mathbf{v}_i + (\partial V_{ij}/\partial \mathbf{x}_j)\mathbf{v}_j$, the equations of motion given by (2) can be writen as

$$\frac{d}{dt}(mv_i^2/2 + \sum_j{}' V_{ij}) = \sum_j{}'(\partial V_{ij}/\partial \mathbf{x}_j)\mathbf{v}_j \ \ , \tag{3}$$

which tells that the rate of change of the one-particle energy $E_i = mv_i^2/2 + \sum_j' V_{ij}$ is the mechanical work made upon the i-th particle per unit time by the rest of the particles. Equation (3) can also be written as

$$dE_i = d(mv_i^2/2 + \sum_j{}' V_{ij}) = \sum_j{}' (\partial V_{ij}/\partial \mathbf{x}_j) d\mathbf{x}_j \ . \tag{4}$$

It is easy to see that equation (4) is not integrable. Indeed, $\partial E_i/\partial x_j = -\partial V_{ij}/\partial \mathbf{x}_j$ and $\partial E_i/\partial x_i = 0$, so $\partial^2 E_i/\partial x_i \partial x_j$ is both vanishing and non-vanishing. It follows that function E_i depends on the integration path, so it is not defined in fact, and it does not exist. The many-body ensembles are not integrable in terms of one-particle quantities. They move as a whole, not as a collection of independent particles, as expected.

Many-particle forces. If we still wish to represent in an approximate way the one-particle motion, then we can introduce a coordinate dependence of the form

$$\delta x_i = g_{ij} dx_j \ , \tag{5}$$

where the matrix g is unknown. In (5) the matrix multiplication must be understood as $\delta x_i^\alpha = g_{ij}^{\alpha\beta} dx_j^\beta$, where α, β stand for the cartesian components of the cordinates. In addition, it is reasonable to assume that the matrix g has the form $g_{ij} = \delta_{ij} + h_{ij}$, where h_{ij} is a symmetric matrix and $h_{ii} = 0$. Obviously, equation (5) can be iterated, so we may write

$$\delta x_i^{(1)} = g_{ij} dx_j, \ \ \delta x_i^{(2)} = g_{ij} g_{jk} dx_k, ... \tag{6}$$

Similar correlations hold for velocities, and the conservation laws must be satisfied to every iteration. We note also that for finite iterations the matrix h must have a less-than-unity norm, so the net contributions to (6) are gradually decreasing with increasing their order. It is a hierarchy.

It is easy to see what is the change in the total potential energy U under the local variations of coordinates given by (6). We get

$$\delta U^{(1)} = \sum_{ijk}{}' (\partial V_{jk}/\partial x_j) g_{ji} dx_i, \ \ \delta U^{(2)} = \sum_{ijkl}{}' (\partial V_{jk}/\partial x_j) g_{jl} g_{li} dx_i, ... \ , \tag{7}$$

so the force acting upon the i-th particle is given by

$$F_i^{(1)} = -\sum_{jk}{}' (\partial V_{jk}/\partial x_j) g_{ji}, \ \ F_i^{(2)} = -\sum_{jkl}{}' (\partial V_{jk}/\partial x_j) g_{jl} g_{li}, ... \ . \tag{8}$$

Making use of $g = 1 + h$, one can see easily the occurrence of three-, four- and higher-order forces, beside the two-particle ones; they can also be represented by schematic graphs, as usually. It follows that we are justified to use the representation of phenomenological many-particle forces and interactions in describing approximately the one-particle motion in a many-body ensemble.

It must however be stressed that these forces are phenomenological. Indeed, an iterative solution of equation (2) (or equivalently (3)) by making use of a suitable mean field is inconsistent (for instance the *rhs* of these equations would become iteratively time-dependent functions and the ensemble would not be conservative anymore). Phenomenological here means that experimental data acquired in terms of one-particle motion can be described conveniently and to a limited extent by many-particle forces with matrix g as fitting parameters.

The relevance. A convenient average of equation (4) over the j-variables leads to a mean field h_i, where the one-particle energy exists and it is represented as $E_i = mv_i^2/2 + h_i$. Such an average is valid for large ensembles. Any other contribution beyond the mean-field picture spoils the notion of one-particle motion. Many-particle forces help to preserve the notion at the price of a phenomenological picture.

In quantum theory the lack of one-particle description is already included since the very beginning in the entangled many-particle wavefunction $\psi(x_1, x_2, ...)$.The Hartree-Fock equations presume the mean field and the independent, though self-consistent, one-particle motion. Consequently, the above limitation is incorporated in the *ansatz* of the theory and finds for itself a fortunate representation with the quasi-particle elementary excitations and their finite lifetime. The self-consistent iterations must be kept consistent with this limitation. This means the first-order iteration. Anything beyond is a one-particle picture with phenomenological fitting parameters of the multi-particle forces.

The situation is similar for composite particles, like pairs in superconductors for instance. There, the particles are subjected to a pair-correlation mean field, the first-order self-consistency is ensured by the gap equation and the quasi-particle lifetime limits successive iterations. The gap parameter becomes itself a fitting parameter for various one-particle properties, which means that multi-particle forces can be incorporated in the pair correlations from the outset. A similar situation holds generically for superfluids too, and for other condensates. This was our basic reason for using phenomenological four-particle forces and interactions in constructing the "four-fermion condensate" in 1984-1986 for nucleons in the atomic nucleus (or bi-excitons in semiconductors).[1] The effect of these four-fermion correlations have been seen in new superfluid states of the atomic nuclei, the rate of the alpha decay, two-nucleon transfer reactions, the fine structure of nuclear cohesion energy, etc.

[1] J. Bulboaca, F. Carstoiu, M. Horoi, M. Apostol, ... under the leadership of O. Dumitrescu

Correspondence: M Apostol, Department of Theoretical Physics, Institute of Atomic Physics, Magurele-Bucharest MG-6, POBox MG-35, Romania, Email apoma@theory.nipne.ro. Received: July 12, 2018 | Published: August 13, 2018. Introduction. In the context of an active topical research in laser-related physics,1â€'5 the problem of charge emission from bound-states under the action of the electromagnetic radiation is receiving an increasing interest. Some investigations focus M. Apostol Department of Theoretical Physics, Institute of Atomic Physics, Magurele-Bucharest MG-6, POBox MG-35, Romania e-mail: apoma@theor1.ifa.ro. J. Theor. Phys.Â Typically,[3] these theories start with an atomic-like Ï€-type orbital for the added electron, and proceed by taking advantage of the neighbouring carbon atoms on the surface of the fullerene molecule for lowering the electron energy. The building blocks of the starting orbital resemble, more or less, the carbon unoccupied pz-atomic orbital. From symmetry requirements it is found that the ground-state of the added electron is triple degenerate and belongs to the t1u-representation of the icosahedral symmetry group of the molecule. Department of Theoretical Physics, Institute of Atomic Physics, MÃ£gurele-Bucharest MG-6, POBox MG-35, Romania email: apoma@theory.nipne.ro. (Received May17, 2006). Abstract. A series expansion in powers of eccentricities is set up for the motion in central-field potentials, which connects such movements to anharmonic oscillators. The method is used to rederive the solution of Keplerâ€™s problem, describe the motion of a particle in a general central-field potential, and discuss the closed orbits and the first sign of chaos. Similarly, Moonâ€™s problem is tackled by the same method, where the motio