A LESSON THAT MAY DEVELOP MATHEMATICAL THINKING OF PRIMARY STUDENTS IN VIETNAM

FIND TWO NUMBERS THAT THEIR SUM AND A RESTRICTED CONDITION ARE KNOWN

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In Vietnam, after launching the national standard mathematics curriculum in 2006, the classroom mathematics teachers have learnt more on the innovative teaching strategies to implement more effective lessons focusing on mathematical thinking. The aim of this paper is to examine a lesson that we considered may develop mathematical thinking of primary students in Vietnam. A case study will be analysed using the observed students' activities in a videotaped lesson.

INTRODUCTION

In Vietnam, teachers encourage their students to invent their own procedures or algorithms for solving problems. The teachers use the teaching strategies that aim to:

- Promote active, initiative and self-conscious learning of the learners;
- Form and develop the ability of self-study;
- Cultivate the characteristics of flexible, independent, and creative thinking;
- Develop and practice the logical thinking;
- Apply problem solving approaches;
- Apply mathematics to real life situations.

In the teachers' guidebook for primary mathematics teachers at each grade there are four main activities in a lesson that teachers should follow to develop mathematical thinking:

Activity 1. Teacher manages students to work and achieve the following aims:

- Examine the students previous knowledge;
- Consolidate the previous knowledge involved with new lesson;
- Introduction to the new lesson.

Activity 2. Teacher facilitates students explore mathematical knowledge and construct new knowledge by themselves.

Activity 3. Students practice the new knowledge by solving exercises and problems in the textbook.

Activity 4. Teacher concludes what students have learnt from new lesson and assigns the homework.

Engaging to the lesson, the pupils will have opportunities to show their mathematical thinking through:

- The ability of observing, predicting, rational reasoning and logical reasoning;
- Knowing how to express procedures, properties by language at specific levels of generalization (by words, word formulas);

- Knowing how to investigate facts, situations, relationships in the process of learning and practicing mathematics;
- Developing ability on analyzing, synthesis, generalization, specifying; and starting to think critically and creatively.

In our point of view, the key windows for considering mathematical thinking are as follows:

- Students learn mathematical concepts with *meaningful understanding*;
- Students construct *individual algorithm* and *techniques* themselves with understanding to solve some specific problems;
- Students use learnt mathematics to *solve* mathematical problems effectively;
- Students show mathematical thinking by *communicating* (talking, writing, arguing, discussing, and representing);
- Students *reflect critically* their mathematical thinking in order to improve their learning;
- Mathematical thinking is *social* and *relative* to each individual student;
- Students apply *logical* and *systematic* thinking in mathematical and other contexts;
- Students use *thinking operations* in solving problems: *comparison*, *analogy*, *generalization*, and *specialization*;

The lesson which will be analyzed in this paper is prepared by classroom teacher for grade five primary students. We can find from the lesson plan the three main tasks and a quiz proposed in the lesson:

> *Introductory Task*. Use 2 cm-cards and 4 cm-cards to make a toy train of 5 wagons?
>
> *Task* 1. Use 2 cm-cards and 4 cm-cards to make a train with the length of 16 cm?
>
> *Task* 2. A train with the length of 50 cm including 20 wagons, how many red wagons and blue wagons are there?
>
> *Task* 3. A train with the length of 100 cm including 36 wagons, how many red wagons and blue wagons are there?
>
> *Quiz* (Homework). There are 33 liters of fish sauce contained in 2 liter-bottles and 5 liter-bottles. The number of bottles used is 12. Find the number of 2 liter-bottles and 5 liter-bottles used. Given that, all bottles are full of fish sauce.

At the end of Grade 4, students know how to solve and express solutions of problems having three operations of natural numbers.

Example. A toy train has 3 wagons with the length of 2 cm, and 2 wagons with the length of 4 cm. Find the length of the train?

Answer. $3 \times 2 + 2 \times 4 = 14$ (cm).

But in the second semester of grade 5, if we set the problem in a reverse way:

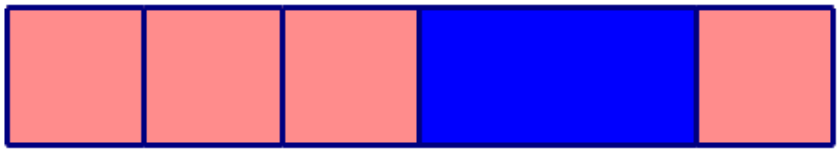
> A toy train has two types of wagon: 2 cm-wagons and 4 cm-wagons. This train has the length of 14 cm including 5 wagons. Find the numbers of 2 cm-wagons and 4 cm-wagons of the train.
>
> The sum of two numbers needed to find is 5.

Restricted condition: The total length of 2 cm-wagons and 4 cm-wagons is 14 cm.

This reverse problem is quite different with what students have learnt in grade 4. The problem requires them to analyze a natural number into sum of two other numbers satisfying a restricted condition logically.

ANALYSIS OF THE TASKS

Analysis of Introductory Task. Use 2 cm-cards and 4 cm-cards to make a toy train of 5 wagons?



This task is an introductory activity. It is an open-ended task that requires pupils to make many trains as possible. Pupils can arrange the cards to make a train, use the strategy "*guess and check*" to get many answers. To solve this task mathematically teacher guides students to make a systematic list of all abilities.

N. of red wagons	0	1	2	3	4	5
N. of red wagons	5	4	3	2	1	0
The length of the train in cm	20	18	16	14	12	10

From the above table, students recognize the relationship between the length and the numbers of red wagons, blue wagons. If the number of red wagons increases one, then the length of the train decreases 2 cm. In this task, students know that:

N. of red wagons + N. of blue wagons = 5

There are 6 options for this task. If the length of the train is given then we can find exactly the N. of red wagons and N. of blue wagons. The length of the train is understood as a restricted condition. Students will see that the train has the longest length 20 cm when all of the wagons are blue and shortest length 10 cm when all of the wagons are red.

The aim of this introductory task is to help students recognize the restricted condition in finding two numbers that their sum is known.

Analysis of Task 1. Make a train with the length of 16 cm.

This is also an open-ended task that requires students to make a systematic list of all abilities. The restricted condition is given but the sum of two numbers is unknown.

N. of red wagons	8	6	4	2	0
N. of blue wagons	0	1	2	3	4

Total of wagons	8	7	6	5	4

There are 5 answers to this task. Students know how to analyze a natural number into sum of two natural numbers with a specific restricted condition.

$16 = 8 \times 2 + 0 \times 4$ $16 = 4 \times 2 + 2 \times 4$ $16 = 0 \times 2 + 4 \times 4$

$16 = 6 \times 2 + 1 \times 4$ $16 = 2 \times 2 + 3 \times 4$

From the table students will see that a train with the length of 16 cm including 6 wagons has 4 red wagons and 2 blue wagons. The restricted condition of this problem is:

Sum: N. of red wagons + N. of blue wagons = 6.

Restricted condition: The length of the train is 16 cm.

If all wagons are red then the length of the train decreases: $16 - 6 \times 2 = 4$, then the number of blue wagons: $(16 - 6 \times 2) \div 2 = 2$.

Students practice this procedure to consolidate what they have learnt. The most important fact that the students need to realize is the difference 2 cm between one blue wagon and one red wagon.

Analysis of Task 2. A train with the length of 50 cm including 20 wagons, how many red wagons and blue wagons are there?

In this task, teacher does not ask students to make a table but encourages them to generalise what they have observed in some concrete situations above to create their own procedure to solve the general problem.

Students make temporary assumption: If the train has only red wagons, the length of the train decreases:

$$50 - 20 \times 2 = 10$$

The number of blue wagons: $(50 - 20 \times 2) \div 2 = 5$.

Students look back the solution by checking their answer: $15 \times 2 + 5 \times 4 = 50$ cm.

Analysis of Consolidation Task. A train with the length of 100 cm including 36 wagons, how many red wagons and blue wagons are there?

The aim of this task is to help students consolidate what they have studied. They use their procedure to solve this problem by using temporary assumption.

The number of blue wagons: $(100 - 36 \times 2) \div 2 = 14$.

Analysis of Quiz. There are 33 liters of fish sauce contained in 2-litter bottles and 5-liter bottles. The number of bottles used is 12. Find the number of 2-liter bottles and 5-liter bottles used. Given that, all of bottles are full of fish sauce.

This is an application task. Students can solve this task as homework. Students learn how to apply what they have studied from the lesson to solve a realistic problem. Students recognise that the difference between one 5-liter bottle and one 2-litter bottle is 3 liters.

The number of 5-liter bottles: $(33 - 12 \times 2) \div 3 = 3$. Thus, the answer is 9 two-liter bottles and 3 five-liter bottles.

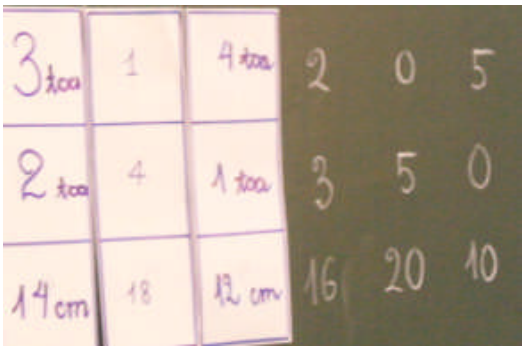
ANALYSIS OF THE VIDEOTAPED LESSON

The lesson is videotaped and analyzed using the video recording and the transcript. The actual lesson included several activities. The analysis in this section will be conducted by dividing the actual lesson into three stages: introductory activities, activities for task 1, and activities for task 2 and task 3. Each stage will be described and analyzed.

Introductory activities

1. Students were asked to make a train of 5 wagons by using 2 cm-red cards and 4 cm-blue cards.
2. Students were asked to use only 5 cards to make their own trains.
3. Students discussed in small groups of 4 students to list as many abilities as possible.
4. Students had to recognize the relationship between the length of the train and the numbers of red wagons, blue wagons.
5. Students had to recognize the restricted condition for each specific case in finding two numbers that their sum is known.
6. From the established table students had to understand that: if the number of red wagons increases one, then the length of the train decreases 2 cm.

In the lesson, some students made only one train of 5 wagons as required and then stop working.



Teacher asked students to paste their answer on blackboard. Most of the answers were presented except the two last options: 5 red wagons and 0 blue wagon, or 0 red wagon and 5 blue wagons

Teacher asked students to arrange the data following a systematic list.

S: There are many answers to this task.

T: Can you check your answer?

S: My train has 5 wagons including 2 red and 3 blue wagons. The length of the train is: $2 \times 2 + 3 \times 4 = 16$ cm.

T: We call "*the length of the train*" the restricted condition. Can you identify another restricted condition?

S: 14 cm.

T: How many red wagons and blue wagons in this train?

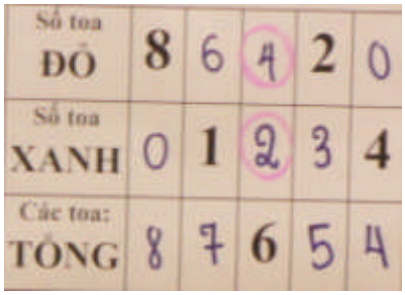
S: 3 and 2. We have $3 \times 2 + 2 \times 4 = 14$ cm.

Activities for Task 1

1. Students were asked to make a train with the length of 16 cm by using 2 cm-red cards and 4 cm-blue cards.
2. Students were asked to use some cards to make their own trains with the same length of 16 cm.

3. Students discussed in small group of 4 students to list as many abilities as possible.
4. Students had to recognize the relationship between the fixed length of the train and the numbers of red wagons, blue wagons.
5. Students had to know to analyse a natural number into sum of two natural numbers with specific restricted conditions.

In this task, students received an almost blank table; some cells have numbers that helped students fill the data into the table easier.



Students analysed number 16 as follows:

$16 = 8 \times 2 + 0 \times 4$ $16 = 2 \times 2 + 3 \times 4$

$16 = 6 \times 2 + 1 \times 4$ $16 = 0 \times 2 + 4 \times 4$

$16 = 4 \times 2 + 2 \times 4$

T: What is given?

S: The length of the train is 16 cm.

T: If the train has 6 wagons, how many red wagons and blue wagons in this train?

S: From the table I saw that this train has 4 red wagons and 2 blue wagons.

T: If we do not make the table, can you explain your solution?

S: If all 6 wagons are red, the train's length decreases 4 cm. So I got 2 blue wagons.

T: Who can express the answer by using mathematical operations?

S: $(16 - 6 \times 2) \div 2 = 4 \div 2 = 2$ (blue wagons).

Activities for Task 2 and Task 3

1. Students were asked to solve an extended problem that is difficult to guess and check.
2. Students were required to create a procedure to solve the task with a specific restricted condition.
3. Students were asked to present their answer by using mathematical operations?

In this task, some students used mental calculations or "*guess and check*" strategy to find out the answers. But they could not explain the answer logically.

S: There are 15 red wagons: $15 \times 2 = 30$ cm. And 5 blue wagons: $5 \times 4 = 20$ cm.

T: I ask you to give a procedure to solve this task not only use your mental calculation.

The teacher guided students to create a procedure by using the temporary assumption to solve the problem.

T: If 20 wagons are red, what is the length of the train?

S: 40 cm.

T: Why does the length decrease?

S: Because we replaced blue wagons by red wagons?

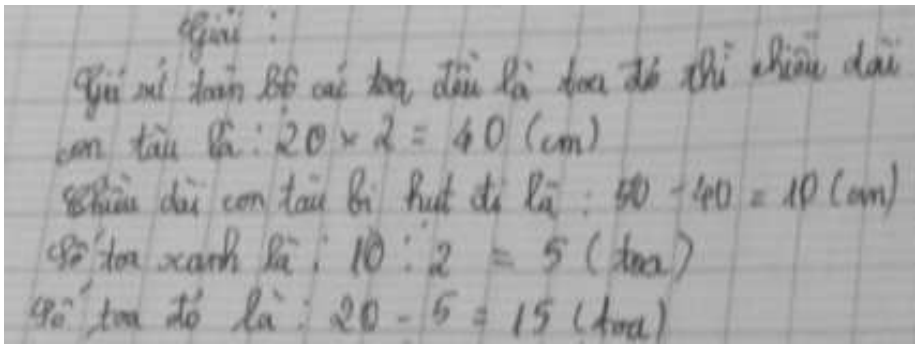
T: How many blue wagons did we replace?

S: 5 blue wagons.

T: How did you get 5?

S: (50 - 40) ÷ 2 = 5.

A student's solution:



Translation into English of a student's solution:

If all wagons are red then the train's length is: 20 × 2 = 40 (cm).

The train's length decreases: 50 - 40 = 10 (cm).

The number of blue wagons: 10 ÷ 2 = 5 (wagons).

The number of red wagons: 20 - 5 = 15 (wagons).

The students applied the procedure to solve Task 3.

The number of blue wagons: (100 - 36 × 2) ÷ 2 = 14 (wagons).

The number of red wagons: 36 - 14 = 22 (wagons).

DISCUSSION AND CONCLUSION

Teaching primary school mathematics aims to equip young pupils with basic mathematics skills and develop their mathematical thinking to solve problems. Some senior classroom teachers have experienced to foster and develop students' mathematical thinking without theoretical background. Most of teachers in Vietnam really need a practical framework to develop pupils' mathematical thinking in their actual classrooms.

This lesson was prepared by a senior teacher, he has involved in some educational projects at primary level. As I analyzed the activities in the lesson by using videotaped recording, the teacher followed four main activities in a lesson that were suggested by the MoET to develop mathematical thinking.

In introductory activities, the task is a open-ended task, it helped students get start to observe many abilities, predict the length of the train by "guess and check". Teacher managed students to work and achieved the following aims:

- Examine the students' previous knowledge in finding the answer for:

  $\square \times 2 + \bigcirc \times 4 = ?$

  where $\square + \bigcirc = 5$.
- Consolidate the previous knowledge involved with new lesson: Find two numbers that their sum is 5. The new lesson needs to have one more restricted condition is the length of the train.

Find □ and ○ such that: □ × 2 + ○× 4 = 14 and □ + ○ = 5.

These activities gave students opportunities to show the ability of observing, predicting, rational reasoning and logical reasoning in solving problems related to the analysis of a natural number into the sum of two other numbers with a restricted condition.

In activities for task 1, teacher facilitated students explore mathematical knowledge and construct new knowledge by themselves. Students recognized the relationship between the fixed length of the train and the numbers of red wagons, blue wagons.

Find □ and ○, where □ × 2 + ○× 4 = 16, but □ + ○ = unknown. Students observed and predicted answers. Students created a procedure to solve the problem when □ + ○ = a fixed number. From specific situations students suggested a procedure to solve general problem. Students invented their own procedures or algorithms for solving problems.

In activities for task 2 and task 3, students practiced the new knowledge by solving exercises and problems given by teacher.

Students applied the analysis of natural number into sum of two other numbers with a restricted condition to solve some mathematics problems systematically by using temporary assumption. These two tasks examined the thinking operations that occurred in the lesson such as: comparison, generalization, and specialization.

Teacher concluded what students have learnt from new lesson and assigned the homework.

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Appendix

Mathematics Lesson Plan

Grade 5 (10-11 years old)

Teacher: Senior Teacher Mr. Tran Quang Khen, Le Qui Don Primary School, Hue City, Vietnam.

1. **Title**: Find two numbers that their sum and a restricted condition between them are known.
2. **About the research theme**
   - Nurturing ability of observing, predicting, rational reasoning and logical reasoning in solving problems related to the analysis of a natural number into the sum of two other numbers with a restricted condition.
   - Examining instruction that focuses on "applying the analysis of natural number into sum of two other numbers with a restricted condition to solve some mathematics problems systematically by using *temporary assumption*".
   - Examining the thinking operations that occur in the lesson such as: comparison, generalization and specialization.

In the national standard mathematics curriculum (2006) for primary level, we emphasize more in word problems that considered being good situations for pupils to explore and solve mathematical problems. Students' mathematical thinking will be enhanced when they solve word problems. Most of these problems are rooted from the real life situations.

At the beginning of Grade 4, students know how to solve and express solutions of problems having three operations of natural numbers.

Example. A toy train has 3 wagons with the length of 2 cm, and 2 wagons with the length of 4 cm. Find the length of the train.

Answer. $3 \times 2 + 2 \times 4 = 14$ (cm).

But if we set the problem in a reverse way:

> A toy train has two types of wagon: 2 cm- wagons and 4 cm - wagons. This train has the length of 14 cm including 5 wagons. Find the numbers of 2 cm- wagons and 4 cm - wagons of the train.
>
> The sum of two numbers needed to find is 5.
>
> Restricted condition: The total length of 2 cm-wagons and 4 cm-wagons is 14 cm.

This reverse problem is quite different with what students have learnt before. The problem requires them to analyze a natural number into sum of two other numbers logically.

3. **Goal**
   - For students to be able to recognize a number as a sum of two other number with a restricted condition;
   - Know how to find two numbers that their sum and a restricted condition are known.

4. Instruction plan

- Understanding the relationship of two related quantities;
- Identifying the restricted condition of the relationship of two quantities.

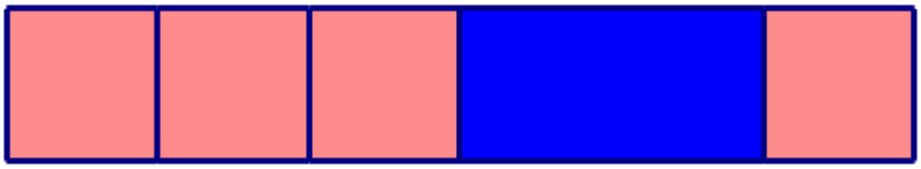
5. Instruction of the lesson

(1) Goal

- For students to realize that making a systematic list will help them understand the problem intuitively;
- Look for a pattern or procedure to solve a set of mathematical problems by using temporary assumption;
- Generalize the procedures obtained to solve some realistic problems;

(2) Flow of the lesson

Teacher prepares some red cards of 2 cm × 2 cm, and some blue card of 4 cm × 2 cm. Teacher called these cards to be wagons of a toy train.



Students are to use cards to make their own train.

Instructional Activities	Points for Consideration
Introductory Task. Use 2 cm cards and 4 cm cards to make a toy train of 5 wagons? List all abilities. How many answers can we get for this problem?	This is an open-ended task that requires students to make a systematic list of all abilities. Guess and check to get many answers.

Students fill the data into the following table.

N. of red wagons		1		3		
N. of blue wagons	5		3	2		0
The length of the train in cm	20					

From the table of data, teacher supports students with following guided questions.

Question 0.1: If the number of red wagons increases one, then what is about the length of the train?	Recognize the relationship between the length and the numbers of red wagons, blue wagons. Recognize the difference between blue wagon and red wagon is 2 cm.
Question 0.2. A train with the length of 14 cm including 5 wagons. How many red wagons and blue wagons are there?	Understand with a restricted condition the answer will be unique.
Question 0.3. When the length of the train is	Identify the restricted condition for

longest? Shortest?	each case.
Task 1. Make a train with the length of 16 cm. List all the abilities and find the number of wagons in your trains?	Know how to analyze a natural number into sum of two natural numbers. Recognize the number of red wagons is an even number. Express the relationship between two quantities: If the number of blue wagons increases one the number of red wagons decreases two. This is also an open-ended task that requires students to make a systematic list of all abilities.

Students fill the data into the following table.

N. of red wagons	8			2	
N. of blue wagons		1			4
Total of wagons			6		

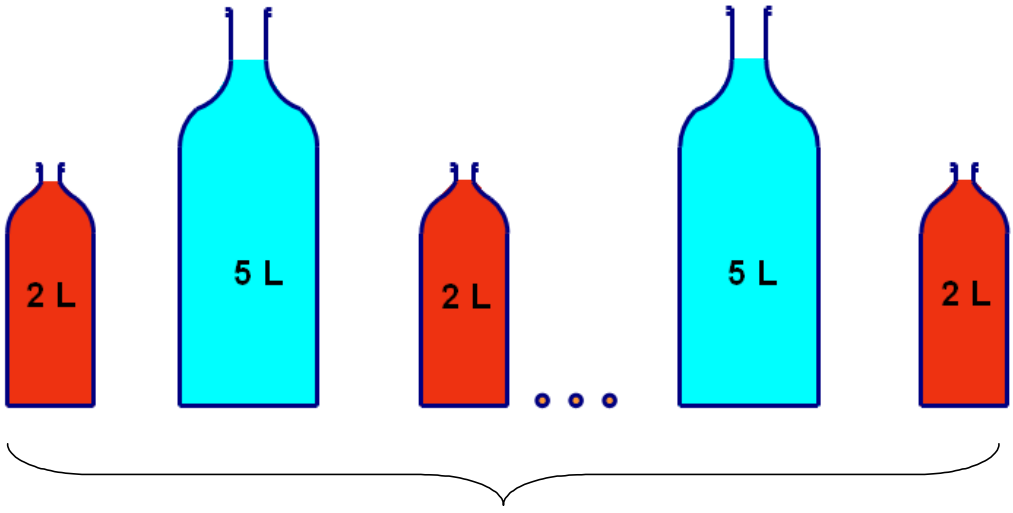
From the table of data, teacher supports students with following guided questions.

Question 1.1: A train with the length of 16 cm including 6 wagons. How many red wagons and blue wagons are there?	Predict the pattern to solve the general problems.
Question 1.2: When does the train have largest number of wagons? Smallest number of wagons?	Identify the restricted condition for each case.

Teacher encourage students create their own procedure to solve the problem.

Task 2. A train with the length of 50 cm including 20 wagons. How many red wagons and blue wagons are there?	The number of blue wagons: $(50 - 20\times2) \div 2 = 5$ Thus, the answer is 15 red and 5 blue wagons.
Question 2.1. If the train has only red wagons, what is the shortened length of the train? *Question* 2.2. In this case, you do not make a table. How can you find the number of blue wagons?	Create a procedure to solve the general problem.
Task 3 (Consolidation Task). A train with the length of 100 cm including 36 wagons, how	The number of blue wagons:

many red wagons and blue wagons are there?	(100 - 36×2) ÷ 2 = 14 Thus, the answer is 22 red and 14 blue wagons. Students practice the procedure just created to solve this problem.
Quiz. There are 33 liters of fish sauce contained in 2-liter bottles and 5-liter bottles. The number of bottles used is 12. Find the number of 2-liter bottles and 5-liter bottles used. Known that all bottles are full of fish sauce.	The number of 5-liter bottles: (33 - 12×2) ÷ 3 = 3. Thus, the answer is 9 two-liter bottles and 3 five-liter bottles. Students apply what they have learnt from the lesson to solve a realistic problem. Asking students to solve another problem to evaluate what the students are learning.



12 bottles containing 33 liters

2 A LESSON THAT MAY DEVELOP MATHEMATICAL THINKING OF PRIMARY STUDENTS IN VIETNAM Dr. Tran Vui Hue University, Vietnam APEC-Khon Kaen International Symposium August 2007. 3 FIND TWO NUMBERS THAT THEIR SUM AND A RESTRICTED CONDITION ARE KNOWN APEC-Khon Kaen International Symposium August 2007 12 bottles containing 33 liters.Â 7 How Do TEACHER DEVELOP STUDENTSâ€™ Mathematical Thinking In a CLASSROOM Setting? (with Teacherâ€™s Background in Solving System of Equations and Beliefs). 8 some backgrounds on the reform of vietnamese education before discussing the research lesson. 9. Researchers found that in teaching English learners -- students who aren't fluent in English and often come from homes where a language other than English is spoken -- the Latin roots of words helped them problem solve the meaning of unfamiliar words. Share: FULL STORY. Lessons in the Latin roots of words may help Spanish-speaking students who are learning English bridge the gap between the two languages. advertisement. In a study, researchers found that teaching English Learners -- students who aren't fluent in English and often come from homes where a language other than English is spoken -- the Latin roots of words helped them problem solve the meaning of unfamiliar words.