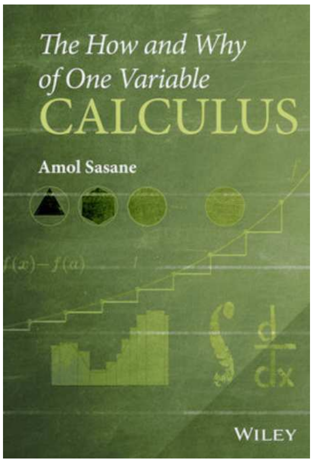
Book Reviews

The How and Why of One Variable CALCULUS

Amol Sasane
Wiley, 2015, ISBN 978-1-119-04338-6

Having taught calculus as a service course for engineering, science and business students for many decades, I can conclude that this book is not for them. However, as a thorough and rigorous approach to the fundamental ideas of the differential and integral calculus for a mathematics undergraduate, it is quite superb. It reminded me of a course that I took at Sydney University many years ago under the late Professor Eric Barnes. I struggled with the fundamental ideas presented, even though I already knew how to differentiate. This book would have been of invaluable assistance to me at the time.

The first three chapters consider real numbers, sequences and continuity in clear detail, expounding on the concepts of sets, rational and irrational numbers, convergence, divergence and limits.



Chapter 4 consists of 58 pages on the basics of differentiation from first principles for each type of function considered. The product rule, quotient rule, chain rule, implicit differentiation, tangent and normal lines, the Newton–Raphson method, local maxima and minima, Cauchy's Generalised Mean Value theorem, Taylor's formula, convexity and l'Hospital's rule are all treated in detail with copious examples and exercises.

One hundred and fourteen pages are devoted to integration in Chapter 5. Here the author covers in detail the Fundamental Theorem of the Calculus, integration of polynomials, algebraic and transcendental functions, integration by parts, Riemann sums and improper integrals. But there is more! Formal definitions of the sine and cosine functions lead to their connection with Fourier Series, while further practical applications of the integral include curves in polar co-ordinates, areas, volumes of solid revolution and arc lengths. Toricelli's Trumpet paradox is also explained clearly.

The final chapter looks at the various tests to prove the convergence of a series, with further detail being given to the analysis of power, Taylor's and Fourier series. But this is not all. Next comes more than one hundred and fifty pages assigned to providing the full solutions to every exercise given in the previous six chapters. These will help any budding pure mathematics student immensely.

The author is from the London School of Economics and has researched his subject very thoroughly with an extensive bibliography. Our only Australian Fields medallist, Terry Tao, receives a mention for the Green–Tao theorem on page 301.

My copy came with four pages of minor errata, and I noted that Descartes and Kronecker were misspelt in the text and index. Nevertheless, this book would be a great text for an initial undergraduate course in pure mathematics and a useful addition to a university library.

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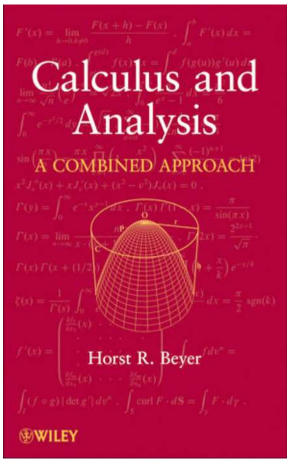
Calculus and Analysis: A Combined Approach

Horst R. Beyer
Wiley, 2010, ISBN 978-0-470-61795-3

The Calculus is a fundamental part of a mathematician's education and every mathematics department will have some sequence of courses named Calculus I, II, and III, or Calculus, Advanced Calculus and Multi-variable Calculus et cetera. Newton's development of Calculus was for the solution of problems of mathematical physics and consequently studying calculus is important in the education of the physicist and the engineer as well.



This large potential audience of students means the marketplace is awash in textbooks for the undergraduate calculus courses. Most texts are large weighty tomes [1] (although there are exceptions [2]) and there exist many resources online including formerly print textbooks digitised and made free [3] and exclusively digital projects [4]. Further, there are many slightly disguised calculus textbooks usually with the words 'engineering mathematics' in the title somewhere which are also typically large volumes and cover the same material, perhaps less rigorously, with a few methods orientated topics thrown in for good measure. The upshot is if you choose to write a calculus textbook you will have a lot of competition.

There is, perhaps, a less populated space existing between the typical undergraduate calculus text and the full-blooded analysis book. A sort of advanced calculus bridge to analysis space and it is this part of the market that Horst Beyer targets with his *Calculus and Analysis: a Combined Approach* (C&A).

Beyer is a professor of mathematics at the University of Michoacán in Mexico. However, his book grew out of lectures he gave at Louisiana State University (LSU), for the calculus I,II,III sequence and he states in his introduction that the book was mainly devised with the entry-level calculus course in mind.

C&A is a hard-backed volume of over 600 pages in length which at the time of writing will set you back approximately $200. The book consists of three parts which follow the sequence of courses from which the text was devised. Part I, after going over logic and some set theory, presents limits and continuity, functions, differentiation and the Riemann integral. Actual techniques of integration are pushed into Part II along with improper integrals, series and the beginnings of vector calculus. The last part covers material from a typical multi-variable calculus course ending at Green's theorem. An appendix details some proofs and provides a construction of the real numbers.

The motivation for Beyer's presentation style is the so-called 'genetic method' of Otto Toeplitz [5]. The concept of the genetic method was to develop mathematics in the classroom in-line with how it evolved historically but without necessarily teaching the history of mathematics. Toeplitz's idea eventually became one volume of a proposed two-volume set published after his death. Beyer claims his book is the first to implement the genetic method and still be able to cover the three-course calculus sequence. Further, applications both historical or otherwise play a big part in motivating the mathematical content.

There are several problems with the book. Firstly, while the content coverage is typical — a standard text such as Stewart [1] actually covers more — the presentation feels terse and at a much higher level than the claimed entry level. Wiley, the publisher of C&A, seem to disagree with the author and state on the back cover that the text is '... an excellent book for ... upper-undergraduate and graduate levels'.

If the publishers are right and this is really a graduate text, and this reviewer tends to think so, then it isn't written in a way that feels like it is a bridge to analysis. Depending on the gradient of the bridge you wish to present students, Zorn's *Understanding Real Analysis*[6] or Alcock's *How to think about Analysis* [7] do a far better job of getting students away from the purely computational thinking of calculus and towards the abstract thinking required for analysis. Further, it is hard to go past Spivak's *Calculus* [8] as a second or advanced calculus book. Interestingly, at the time of writing, LSU use a later edition of Stewart as their compulsory text for the calculus sequence of courses that C&A is derived from.

Next the claimed use of history (or genetic method) and applications feels somewhat overblown. There are a few snippets of historical facts or motivations for a given problem or approach but they are easily lost in the reading and one doesn't get a feeling of development as the genetic method intends. Nor is C&A unique in using a historical motivation [9]. Beyer and the reviewer have different conceptions of applications. Most of the applications of the calculus presented in C&A are to other areas of mathematics such as geometry. Many applications are pushed into the exercises and don't feature prominently in the text. The applications that do come from outside mathematics are entirely from the physical sciences and are the same ones to be found in almost any calculus book. The one exception was an application to probability and boson/fermion statistics.

Lastly, the figures in the book seem not to have consistent fonts. Most of the surface plots were obviously originally created using many colours. They have not been adapted for black and white printing and thus the figures appear dark on the page.

If one doesn't have an advanced calculus text then Beyer's *Calculus and Analysis: A Combined Approach* **could** be useful. It covers quite a bit of material in a compact hard-cover format. But I would not recommend it for undergraduate classes. Indeed, both the professional and the student, aren't lacking for better alternatives which cost significantly less.

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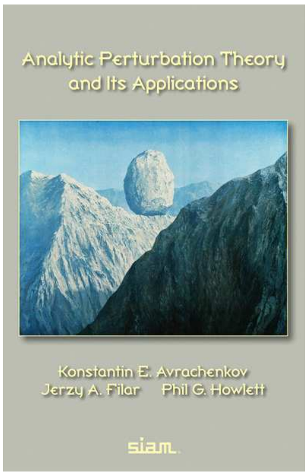
Analytic Perturbation Theory and its Applications

Konstantin E. Avrachenkov, Jerzy A. Filar and Phil G. Howlett
SIAM, 2013, hardcover, xii+372pp, $89.00, ISBN 978-1-61197-313-6

The book is devoted to a study of the general linear and nonlinear systems of algebraic equations depending on a small perturbation parameter. While the coefficients of the equations are assumed to be expandable as power series of the perturbation parameter, their solutions may have more complicated expansions such as Laurent or even Puiseux series, the latter forming a basis for the asymptotic analysis (as the perturbation parameter tend to zero). This analysis is applied in the book to a wide range of problems including constrained optimization, Markov processes and linear operators on Hilbert and Banach spaces.



In a vast variety of applications, system of governing equations include parameters, the values of which are known only approximately (up to a certain level of accuracy), and it is critically important to understand how small deviations from these nominal values may affect solutions of the governing equations. This stimulated the development of the perturbation theory which, due to its importance for applications and due to the mathematical challenges it is presenting, has attracted the attention of many researchers worldwide. In fact, numerous papers and a good number of books have been devoted to different aspects of the perturbation theory (see an overview in the book).

The overarching theme of the book is the asymptotic behavior of solutions to the perturbed systems of equations

$$f(x,\varepsilon) = 0, \tag{1}$$

where $\varepsilon > 0$ is a small parameter and $f(x,\varepsilon)$ is a system of linear or polynomial equations. An important special case of (1) is the system of perturbed linear equations

$$A(\varepsilon)x = b(\varepsilon), \tag{2}$$

where $A(\varepsilon)(n \times n)$ and $b(\varepsilon)(n \times 1)$ are analytic matrix functions of ε, the analysis of (2) relying on the fact that $A^{-1}(\varepsilon)$ can always be expanded as a Laurent series

$$A^{-1}(\varepsilon) = \frac{1}{\varepsilon^s}B_{-s} + \cdots + \frac{1}{\varepsilon}B_{-1} + B_0 + \varepsilon B_1 + \cdots . \tag{3}$$

The system (2) is called regularly perturbed if $B_{-s} = \cdots = B_{-1} = 0$ and $B_0 = A(0)^{-1}$. It is called singularly perturbed if some of the matrix coefficients corresponding to negative powers of ε are not zero, this situation occurring when $\det(A(0)) = 0$.

A significant part of the book is devoted to studying singularly perturbed linear systems in both finite and infinite dimensional spaces (with $A(\varepsilon)$ being interpreted as a linear operator acting on Hilbert or Banach spaces respectively). A significant part of the book is also devoted to applications of singularly perturbed systems in Asymptotic Simplex method, in Markov chains and in Markov decision processes. These parts of the book are closely related and complementary to similar topics dealt with in Baumgartel [1], Kato [3], Pervozvanski and Gaitsgory [5], Schweitzer [6] and Yin and Zhang [7].

In a more general case when the perturbed system is nonlinear (as in (1)), its solution $x(\varepsilon)$ may not be expandable into the Laurent series but, provided that the nonlinearity is polynomial, it is expandable into the Laurent–Puiseux series.

The latter involves fractional powers of the parameter, which may be both positive and negative. Like in the linear case, the presence of negative (fractional or integer)powers in the Laurent–Puiseux series of the solution makes the system singularly perturbed.

An important part of the book is devoted to the theory of Laurent–Puiseux expansions of solutions of nonlinear perturbed systems and their applications in perturbed optimization problems. One important instance of such application is the perturbed mathematical programming problem

$$\max\{r(y,\varepsilon)\} \tag{4}$$

$$\text{s.t. } g_i(y,\varepsilon)=0, \qquad h_j(y,\varepsilon)\le 0; \qquad i=1,\dots,m,\ j=1,\dots,p, \tag{5}$$

where $y\in\mathbb{R}^n$, $\varepsilon\in[0,\infty)$ and $r(\cdot),g_i(\cdot),h_j(\cdot)$ are polynomial functions on $\mathbb{R}^n\times[0,\infty)$ (note that the system of equations (1) is formed in this case with the help of necessary/sufficient optimality conditions). This class of perturbed problems is closely related to the well-established topics of sensitivity or parametric analysis of mathematical programs (see Bonnans and Shapiro [2] and Levitin [4]). However, the approach developed in the book allows one to deal with singularly perturbed problems, the latter may occur even in the well understood linear programming setting.

Consider, for instance, the following elementary linear programming problem in two variables

$$\max_{y_1,y_2}\{y_2\} \qquad \text{s.t.}$$

$$y_1+y_2=1, \qquad (1+\varepsilon)y_1+(1+2\varepsilon)y_2=1+\varepsilon, \qquad y_1,y_2\ge 0.$$

For any $\varepsilon>0$, the problem has a unique (and, hence, optimal) feasible solution $y_1(\varepsilon)=1$, $y_2(\varepsilon)=0$. The corresponding dual problem is of the form

$$\min_{\lambda_1,\lambda_2}\{\lambda_1+(1+\varepsilon)\lambda_2\} \qquad \text{s.t.}$$

$$\lambda_1+(1+\varepsilon)\lambda_2\ge 0, \qquad \lambda_1+(1+2\varepsilon)\lambda_2\ge 1.$$

It also has a unique optimal solution $\lambda_1(\varepsilon)=-\frac{1}{\varepsilon}-1$, $\lambda_2(\varepsilon)=\frac{1}{\varepsilon}$. The appearance of negative power of ε in the presentation of the optimal dual solution makes the problem singularly perturbed, which, in particular, is highlighted by the fact that the optimal value function $F^*(\varepsilon)$ is discontinuous at $\varepsilon=0$,

$$0=\lim_{\varepsilon\to 0}F^*(\varepsilon)\ne F^*(0)=2.$$

The book is organized as follows. It consists of three parts. Part I contains chapters dealing with final dimensional perturbations in both linear and nonlinear cases. Part II consists of chapters devoted to applications of the perturbation analysis in Optimization, Markov Chains and Markov decision Processes. Part III includes chapters devoted to perturbations of linear operators in infinite dimensional spaces. Along with new results, the book contains a lot of expository material, examples and problems which makes a basis for using it as a textbook for graduate and undergraduate courses.

Let us conclude by noting that the material of the book is very well presented, the results are stated in a most clear way, the proofs are rigorous and the bibliography is adequate. There is no doubt that this book is an excellent addition to the existing literature on the perturbation theory and its application and that it will stimulate further research in the area.

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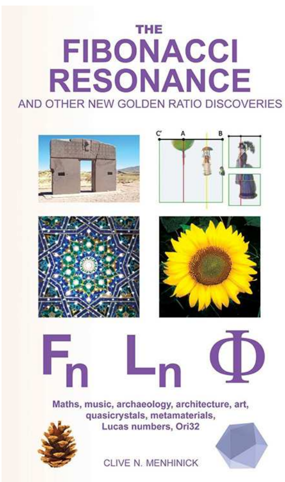
The Fibonacci Resonance

Clive N. Menhinick



onal Ltd, 2015, ISBN 978-0-993-21660-2

he Fibonacci Iteration Numbers:

$$
\begin{array}{ccccccccccccc}
1 & + & 1 & = & 2 & & & & & & & & \\
 & & \downarrow & & \downarrow & & & & & & & & \\
 & & 1 & + & 2 & = & 3 & & & & & & \\
 & & & & \downarrow & & \downarrow & & & & & & \\
 & & & & 2 & + & 3 & = & 5 & & & & \\
 & & & & & & \downarrow & & \downarrow & & & & \\
 & & & & & & 3 & + & 5 & = & 8 & & \\
 & & & & & & & & \downarrow & & \downarrow & & \\
 & & & & & & & & 5 & + & 8 & = & 13 \quad \cdots
\end{array}
$$

Like the role played by the properties of the prime factorization of numbers in number theory, the properties of Fibonacci numbers and the Golden Ratio play a fundamental role in understanding the patterns of nature and human creativity, the proof of which is the *raison d'être* of this book by Clive Menhinick.

The book is a kaleidoscopic dictionary of pictures, diagrams and mathematical relationships with a Fibonacci connection, such as the Golden Ratio in architecture, the phyllotaxis of the leaves and flowers of plants, the appeal in music of the well ordering of the Fibonacci numbers lacking of periodicity and regularity, and the mathematics of Binet and Lukas numbers and the author's Ori32 geometry. Other number patterns are discussed including kissing numbers.

An interesting aspect of the book is how the author, in an innovative manner, has connected properties of Fibonacci numbers with historical and current scientists, musicians and personalities. Bartók's fascination with nature, the Golden Ratio and Fibonacci numbers is discussed along with attempts by various authors (Lowman) and musicologists (Lendvai) to formalize connections between some of Bartók's compositions and Fibonacci numbers. The use by Bartók in his Music for Strings, Percussion and Celesta of the Fibonacci sequence in the Xylophone solo. Because they worked in Paris at the same time, Menhinick catalogues, on pages 149–151, the activities of Lukas, Henry, Seurat, Debussy, Ravel and Bartók in order to speculate about how these musicians may have had contact with and been influenced by the pattern and magical appeal of the Fibonacci numbers.

Various historical and current mathematicians, scientists and personalities are mentioned, although some have only a tenuous link with the focus of the book. For example, Pythagoras is mentioned in relation to the five-pointed star within a regular pentagon with crossing chords each dividing the other in the Golden Ratio; Kepler and Newton, via the *kissing numbers* associated with the packing of spheres; Faraday, via being admired by Einstein; Einstein, via his concern about the validity of quantum mechanics, via his appreciation of *Le Corbusier*'s utilization of the Golden Ratio in architecture and his book *The Modular*; Turing, via his attempts (unpublished) in later life to explain 'the appearance of Fibonacci numbers in nature'; and Feynman, via his views about wave-particle dynamics. With respect to Turing's unpublished attempts, a detail account is given in Swinton's Chapter, entitled 'Watching the daisies grow: Turing and Fibonacci phyllotaxis', in the book *Alan Turing: Life and Legacy of a Great Thinker* [1].

The role of the Golden Ratio in architecture and art is quite explicit, with Menhinick giving many pictures and diagrams. He includes, on page 71, a list of artists, composers and architects and the sources for their mathematical knowledge.

There are many snippets of surprising, insightful and informative detours, which a reader can explore, including:

(i) Interesting footnotes, such as the one on page 125 about the Golden Ratio (1.609 . . .) being a more appropriate conversion formula for miles to kilometers than the Fibonacci ratio 8/5. Consequently, a table of Fibonacci numbers could be used in an additive manner to do the conversions from miles to kilometers and kilometers to miles.

(ii) The connection of the Golden Ratio with fractals. The earlier works of Helge von Koch, Piet Mondrian and Lewis Fry Richardson (Richardson is not listed in the Index), that predate the fundamental conceptualizations and publications of Benoit Mandelbrot, are acknowledged. The connection of various fractal patterns with the concept of fractal dimension is explained.

(iii) The original research by the author in terms of listing interesting properties and connections related to his Ori32 geometry and number system.

(iv) The energy minimization experiment of Cristiano Nisoli with the magnetic cactus which showed phyllotactic patterns.

(v) The levitation of a small magnet above a superconductor (the Meissner effect).

(vi) The connection of the Golden Ratio with the pyramids. This is an example of the 'arithmetic games' played by various authors over the years, who aimed to show that the Egyptians used approximations of π and the Golden Ratio in the construction of the pyramids. It is an example of some, but not many, of the tenuous assertions in the book.

(vii) Leonardo da Vinci's involvement with the branching of trees. The source cited (Minamino and Tanto [2]) has the title 'Tree branching: Leonardo da Vinci's Rule versus biomechanical models'. Leonardo da Vinci specifically hypothesized that the sum of the cross-sectional area of all tree branches above a branching point at any height is equal to the cross-sectional area of the trunk. Minamino and Tanto discuss in some detail circumstances under which the rule is valid and how it relates to biomechanical models.

(viii) Many pages are devoted to an informative and insightful discussion about the hypothesizing by Penrose that, on the possibility that viruses grow as Ammann non-periodic solids, crystals could grow quasi-periodically, Mackay's prediction of quasicrystals and their subsequent discovery by Shechtman. This discussion, in order to give the relevant background, detours into tiling patterns and electron diffraction, and surmises about hidden occurrences of the Golden Ratio in the diffraction patterns of quasicrystals.

The book appeals to and stimulates our subconscious quest for 'the magic of patterns'. Readers fascinated with the interrelationship between numbers and patterns of nature will certainly enjoy it.

Many of the things that could have been said by this reviewer about the mathematics in this book have already been said informatively in the following review: http://www.euro-math-soc.eu/review/fibonacci-resonance-and-other-new-golden-ratio-discoveries. It includes a detailed explanation of the meaning of the key words in the title 'Fibonacci Resonance' and their connection to the author's Ori32 geometry.

A useful aspect of the book, which gives it a dictionary character, are the excellent comprehensive Appendices making up about one-third of the book: glossary, symbols used, collected formulas, references to publications and to specific items in publications (1004), about the author, list of figures, and a comprehensive index, which allows one to explore further connections which come to mind when reading about some particular topic. In a way, it is not only a book that can be read from cover to cover. It is also one that can be used to explore personal predilections related to Fibonacci numbers, the Golden Ratio in terms of connections with historical and current mathematicians, scientists and musicians, and with nature, architecture and science, which includes connections with quantum mechanics, metamaterials, quasicrystals and fractals.

The book is lucidly written without obvious errors. It is sometimes quite enthusiastic about the historic discoveries of the Golden Ratio. There are indexing errors (Mandelbrot 95 appears in page 96; the chapter on quasicrystals starts on page 323 not 318, though quasicrystals are mentioned on page 318 in relation to Penrose tilings). This is not surprising given the size and complexity of the cross-referencing of topics and people.

There are different ways in which this book will appeal to a wide readership. As well as the possibilities mentioned above, it can be used as a source for elementary mathematical examples, diagrams and pictures which have various connections to the real world, musicians, mathematicians and scientists, to stimulate the curiosity of children, students, colleagues and the wider community.

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Calculus of Variations with Many Variables. We've found the equations defining the curve along which the integral. has a stationary value, and we've seen how it works in some two-dimensional curve examples. But most dynamical systems are parameterized by more than one variable, so we need to know how to go from a curve in to one in a space , and we need to minimize (say). In fact, the generalization is straightforward: the path deviation simply becomes a vector, Then under any infinitesimal variation (writing also.) Just as before, we take the variation zero at the endpoints, and in How can we use first and second derivatives to locate local minima and maxima, and why does this work? (You should be able to write out the complete second derivative test by thinking about functions, without memorizing anything or consulting any notes.) Why is the fundamental theorem of calculus true? Each of these is much more important than the sorts of things you're actually likely to be tested on in a calculus class, such as: If [math]f(x) = \sin(\cos(\tan(x))[/math], what is [math]f''(x)[/math]? What is the indefinite integral [math]\int \sin^3(x) \, dx[/math]? Someone who Calculus, originally called infinitesimal calculus or "the calculus of infinitesimals", is the mathematical study of continuous change, in the same way that geometry is the study of shape and algebra is the study of generalizations of arithmetic operations. It has two major branches, differential calculus and integral calculus; the former concerns instantaneous rates of change, and the slopes of curves, while integral calculus concerns accumulation of quantities, and areas under or between curves