Depth of field and Scheimpflug's rule : a "minimalist" geometrical approach

Emmanuel BIGLER

September 6, 2002

ENSMM, 26 chemin de l'Epitaphe, F-25030 Besancon cedex, FRANCE, e-mail : bigler@ens2m.fr

Abstract

We show here how pure geometrical considerations with an absolute minimum of algebra will yield the solution for the position of slanted planes defining the limits of acceptable sharpness (an approximation valid for distant objects) for Depth-of-Field (DOF) combined with SCHEIMPFLUG's rule. The problem of Depth-of-Focus is revisited using a similar approach. General formulae for Depth-Of-Field (DOF) are given in appendix, valid in the close-up range. The significance of the circle of least confusion, on which all DOF computations are based, even in the case of a tilted view camera lens and the choice of possible numerical values are also explained in detail in the appendix.

Introduction

We address here the question that immediately follows the application of SCHEIMPFLUG's rule: when a camera is properly focused for a pair of object/image conjugated slanted planes (satisfying "SCHEIMPFLUG's rules of 3 intersecting planes"), what is actually the volume in object space that will be rendered sharp, given a certain criterion of acceptable sharpness in the (slanted) image/film plane?

Again the reader interested in a comprehensive, rigorous, mathematical study based on the geometrical approach of the circle of confusion should refer to Bob WHEELER'S work [1] or the comprehensive review by Martin TAI [2]. A nice graphical explanation is presented by Leslie STROEBEL in his reference book [5], but no details are given. We re-compute in the appendix STROEBEL'S DOF curves and show how they are related to the classical DOF theory. The challenge here is to try and reduce the question to the absolute minimum of maths required to derive a practical rule.

It has been found that, with a minimum of simplifications and sensible approximations, the solution can be understood as **the image formation through the photographic lens fitted with an additional positive or negative "close-up" lens** of focal length $\pm H$, where H is the hyperfocal distance. This analogy yields an immediate solution to the problem of depth of field for distant objects, the same solution as documented in Harold M. MERKLINGER's work [3], [4], which appears simply as an approximation of the rigorous model, valid for far distant objects.

1 Derivation of the position of slanted limit planes of acceptable sharpness

1.1 Starting with reasonable approximations

Consider a situation where we are dealing with a pair of corresponding slanted object and image planes according to SCHEIMPFLUG's rule (fig. 4), and let us first assume a few reasonable approximations:

1. first we neglect the fact that the projection of a circular lens aperture on film, for a single, out of focus point object, will actually be an *ellipse* and *not* a circle. This is well explained by Bob WHEELER [1] who shows, after a complete rigorous calculation, that this approximation is very reasonable in most practical conditions.

2. second we consider only far distant objects; in other words we are interested to know the position of *limit surfaces* of sharpness *far* from the camera, i.e. distances s or p much greater than the focal length f. We'll show that those surfaces in the limit case are *actually* planes, the more rigorous shape of these surfaces for all object-to-camera distances can be found in Bob WHEELER's paper, in Leslie STROEBEL's book [5], and here in the appendix.

3. finally we'll represent the lens as a single positive lens element; in other words we neglect the distance between the principal planes of the lens, which will not significantly change the results for far distant objects, provided that we consider a quasi-symmetrical camera lens (with the notable exception of telephoto lenses, this is how most view camera lenses are designed).

1.2 A "hidden treasury" in classical depth-of-field formulae !

Let us restart, as a minimum of required algebra, with the well-know expressions for classical depth of field distances, in fact the ones used in practice and mentioned in numerous books, formulae on which are based the DOF engravings on classical manually focused lenses.

Consider an object AB perpendicular to the optical axis, let p_1 and p_2 the positions (measured from the lens plane in O) of the planes of acceptable sharpness around a given position of the object p.

It should be noted (see fig. 1) that the ray tracing for a couple of points outside the optical axis like D_1 and D'_1 yields in the image plane $A'B'$ an out-of-focus image of *circular* shape (not an *ellipse*, as it could be considered at a first); this out-of-focus image is exactly the same as the circular spot originated from P_1; this is simply the classical property of the conical projection of a circular aperture between two parallel planes. The out-of-focus spot near D' is centred on the median ray $D\,D_1\,O\,D'\,D'_1$ that crosses the lens at its optical centre. This point will be important in the discussion about transversal magnification factors for out-of-focus images.

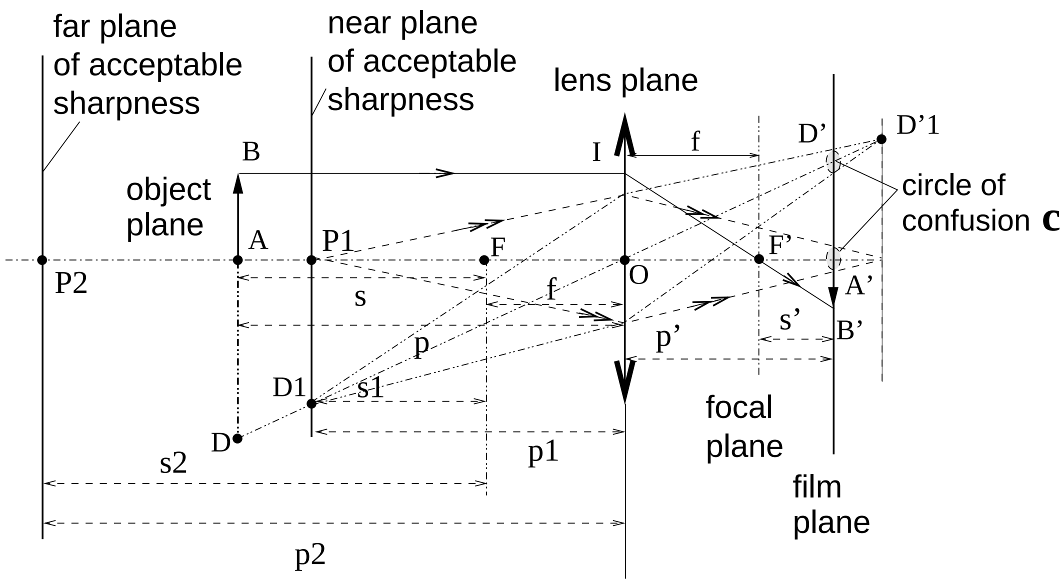


Figure 1: Depth-of-field distances s_1, p_1 and s_2, p_2 for a given circle of confusion c

Assuming a given value for c, the diameter of the circle of confusion, p_1 and p_2 are identical whether we consider AA' on-axis or DD' off-axis and is given, for far distant objects, by

$$\frac{1}{p_1} = \frac{1}{p} + \frac{1}{H}\ ;\ \frac{1}{p_2} = \frac{1}{p} - \frac{1}{H} \tag{1}$$

In eq.(1), H is the **hyperfocal** distance for a given numerical aperture N and diameter of the circle of confusion c, defined as usual as

$$H = \frac{f \times f}{N \times c} = \frac{f^2}{Nc} \tag{2}$$

The previous expressions of eq. (1) are valid only for far-distant objects; readers interested by more exact expressions, valid also for close-up situations, will find them below in the appendix.

Now let us combine eq.(1) with the well-known object-image equation (known in France as DESCARTES formulae), written here with *positive* values of p and p' (the photographic case)

$$\frac{1}{p} + \frac{1}{p'} = \frac{1}{f} \tag{3}$$

Combining equation (1) into (3) yields the interesting formula (4)

$$\frac{1}{p_1} + \frac{1}{p'} = \left(\frac{1}{f} + \frac{1}{H}\right)\ ;\ \frac{1}{p_2} + \frac{1}{p'} = \left(\frac{1}{f} - \frac{1}{H}\right) \tag{4}$$

which is nothing but the object-image equation for the image A' located at a distance p' of the lens centre O, but as seen through an optical system of inverse focal length $\left(\frac{1}{f}+\frac{1}{H}\right)$ for the near plane p_1 (point P_1) and $\left(\frac{1}{f}-\frac{1}{H}\right)$ for the distant plane p_2 (point P_2). The expression $\left(\frac{1}{f}+\frac{1}{H}\right)$ is simply the inverse $\frac{1}{f_1}$ of the focal length of a compound system made of the original lens, fitted with a positive close-up lens of focal length H.

When you "glue" two thin single lens elements together into a *thin compound with no air space*, their *convergences* (inverse of the focal length) should simply be added. Thus in a symmetric way $\left(\frac{1}{f}-\frac{1}{H}\right)$ is nothing but the inverse $\frac{1}{f_2}$ of the focal length of a compound system made of the original lens, fitted with a *negative* "close-up" lens of focal length $-H$.

1.3 A positive or negative "close-up" lens to visualise DOF at full aperture?

1.3.1 a classical DOF rule revisited

Before we proceed to the SCHEIMPFLUG case, let us examine the practical consequence of the additional close-up lens approach in the simple case of parallel object and image planes. We'll show how this additional lens element will allow us to revisit some well-know DOF rules (figure 2).

It is known to photographers that when a lens is focused on the hyperfocal distance H, all objects located between $H/2$ and infinity will be rendered approximately sharp on film, i.e. sharp within the DOF tolerance. Consider a lens focused on the hyperfocal distance and let us add a positive close-up lens element of focal length $+H$. The ray tracing on figure 2 shows that the object plane located at a distance $H/2$ is now imaged sharp on film if the lens to film distance is unchanged. In a symmetric way, the same photographic lens fitted with a negative "close-up" additional lens element of focal length $-H$ will focus a sharp image on film for objects located at infinity. The same considerations actually apply whatever the object to lens distance might be, in this case formulae (4) are simply a more general rule valid for any object to lens distance p; the result is eventually the same, i.e. the positions of acceptable sharpness p_1 and p_2 are located where the film "would see sharp" through the camera lens fitted with a positive or a negative "close-up" lens of focal length $+H$ or $-H$.

1.3.2 DOF visualisation at full aperture??

It would be nice to be able to use this trick in practice to check for depth of field without stopping the lens down to a small aperture. In large format photography, f/16 to f/64 are common, and the brightness of the image is poor; it is difficult to evaluate DOF visually in these conditions. The close-up lens trick would, in theory, allow to visualise the positions of limit planes of acceptable sharpness at full aperture simply by swapping a $+H$ or $-H$ supplementary lens by hand in front of the camera lens with the f-stop kept wide open.

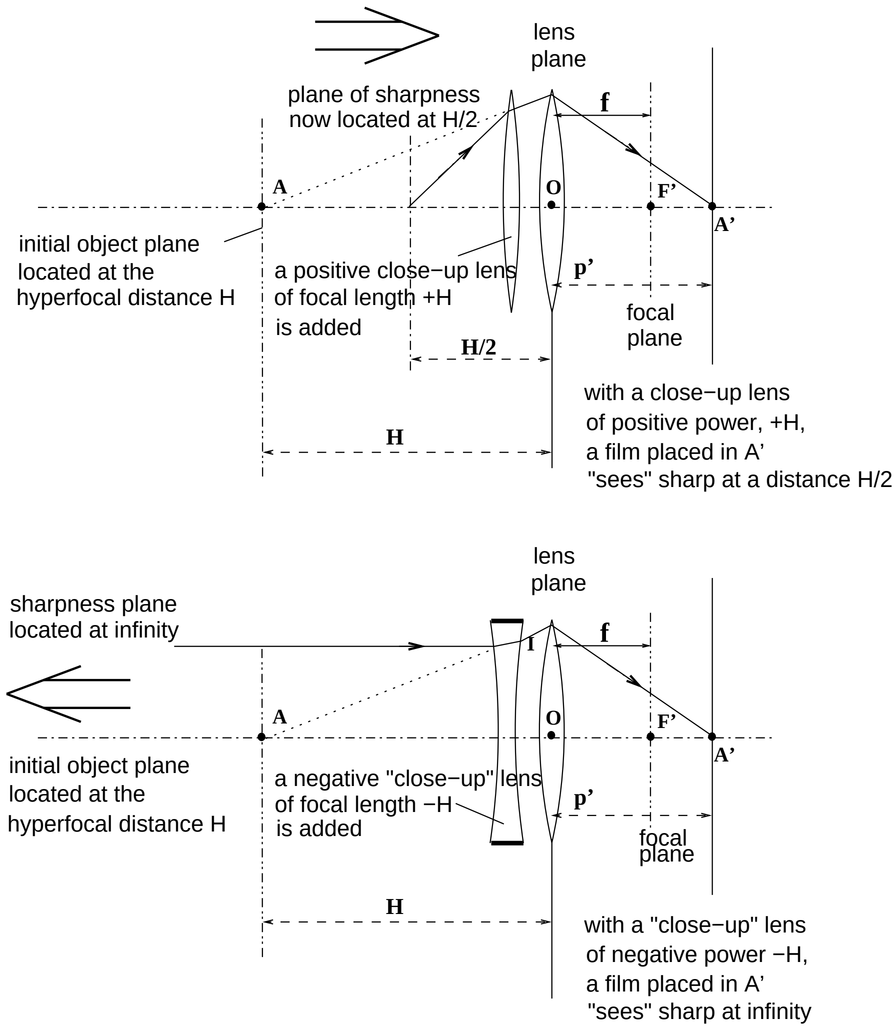


Figure 2: When the an additional "close-up" lens of positive or negative power +H or -H allows us to find well-know DOF rules

There is no reason why this could not work from an optical point of view. Unfortunately classical close-up lenses are always *positive* and, to the best of my knowledge, my favourite opticist round the corner will not have in stock eyeglasses with a focal length longer than 2 metres (a power smaller than 0.5 dioptre). The shop will probably have all kinds of positive and negative lenses in stock, but none, even on special order, will exhibit, say, a focal length of 10 metres (1/10 dioptre) because this is quite useless for correcting eyesight.

In large format photography, the hyperfocal distance is always greater than 2 metres. For example a large format lens, with a focal length of 150 mm, for which we consider appropriate a circle of confusion of 100 microns has an hyperfocal distance smaller that 2 metres only when closed down at a f-stop smaller than f/112. An impossible aperture, and moreover for such "pinhole" kind of values, diffraction effects make the classical DOF model questionable.

Let us however see if this could work in 35mm photography. Shift and tilt lenses do exist for 35mm SLR cameras. Consider a moderate wide-angle of 35 mm focal length and assume that the circle of confusion is chosen equal to the conventional and widely used value of 33 microns. The hyperfocal distance is equal to 2 metres at f/18: this is a more realistic value. Those who use shift and tilt lenses, 35mm focal length on a 35mm SLR can actually use a +0.5 or -0.5 dioptre supplementary lens to get an idea of the DOF planes at f/16-f/22 without actually stopping down the lens. And we'll show below that this will be useful also *for moderate tilt angles*.

For large format photographers, actually the majority of users of tilts and shifts, the trick of a positive or negative "close-up" lens will only be a very simple geometrical help to determine where the slanted planes of acceptable sharpness in object space are located, as explained now.

1.4 Where Mr. Scheimpflug helps us again and gives the solution

When the film plane is tilted, the ray tracing is similar to the one on figs.1 and 2, but the object plane is slanted (figures 3 and 4). We show now that even in this case, we can also consider the camera lens fitted with a positive or negative close-up lens to determine the object planes of acceptable sharpness.

1.4.1 a last argumentation without analytical calculations...

Now a subtle question that arises is: we now have the formula connecting the *longitudinal* position of out-of-focus pseudo-images (actually: elliptical patches, close to a circle, when the tilt angle is small) in the slanted film plane with the corresponding longitudinal position of a point source in the object space. But what is the *transversal* magnification factor? To find this we need an additional diagram (figure 3).

Due to basic properties of a geometrical projection of centre O, *if we neglect the "ellipticity" of the DOF spot*, the centre of the out-of-focus image, A_2'', is aligned with the median ray $A_2\,O\,A_2'$. Hence, the transversal magnification factor for an out-of-focus image A_2'' is *the same as a for a true image* when A_2'' is formed "sharp" through a compound lens fabricated by adding a thin

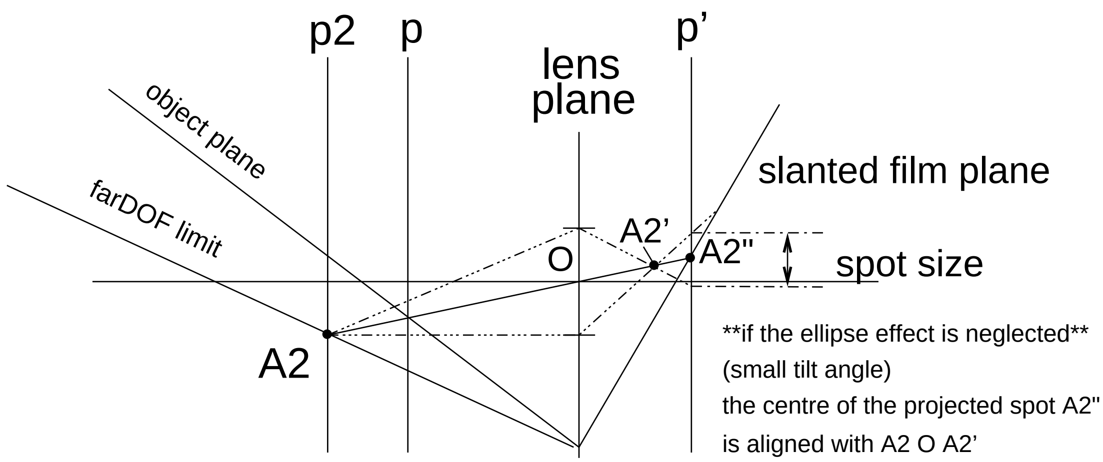


Figure 3: The transversal magnification factor for an out-of-focus pseudo-image, p'/p_2, is the same as for a true image through a compound lens

supplementary lens to the camera lens. This transversal magnification factor (see fig. 3) is equal to p'/p_2, the same value would be obtained for A_2'' as a true image through the compound lens.

So both in *longitudinal* and *transversal* position the correspondence between the object space and the image space for out-of-focus images is exactly the same as if viewed "sharp" through the compound lens. Applying basic rules of *true object-image formation* we already know that the image of a slanted plane is another slanted plane, we do not need any analytical proof to derive what follows.

1.4.2 ...and Mr. Scheimpflug gives us the solution without any calculation!!

As a consequence, without any further calculations we apply SCHEIMPFLUG's rule to the *compound* optical system and **we conclude that the limit surfaces of acceptable sharpness for distant objects are the slanted conjugate planes of the film plane with respect to a compound, thin lens centred in O, with a focal length equal to f_1 (for p_1) or f_2 (for p_2), and that all those planes G_1P_1 and G_2P_2 intersect together in S with the slanted object plane AS and the slanted SCHEIMPFLUG-conjugated image plane SA' as on fig.4.**

To actually define where those planes are located, we simply have to impose that they should cut the optical axis at a distance p_1 (point P_1) or p_2 (point P_2), respectively. Then, simple geometric considerations on homothetic triangles P_2AB_2 vs. P_2OS as well as B_1AP_1 vs. SOP_1 combined with eq.1 yield the interesting and most simple final result: with $h = OS$, both distances h_1 and h_2 are equal to

$$h_2 = h_1 = h\frac{p}{H} \tag{5}$$

Now consider a plane G_1GG_2 perpendicular to the optical axis and located at the hyperfocal

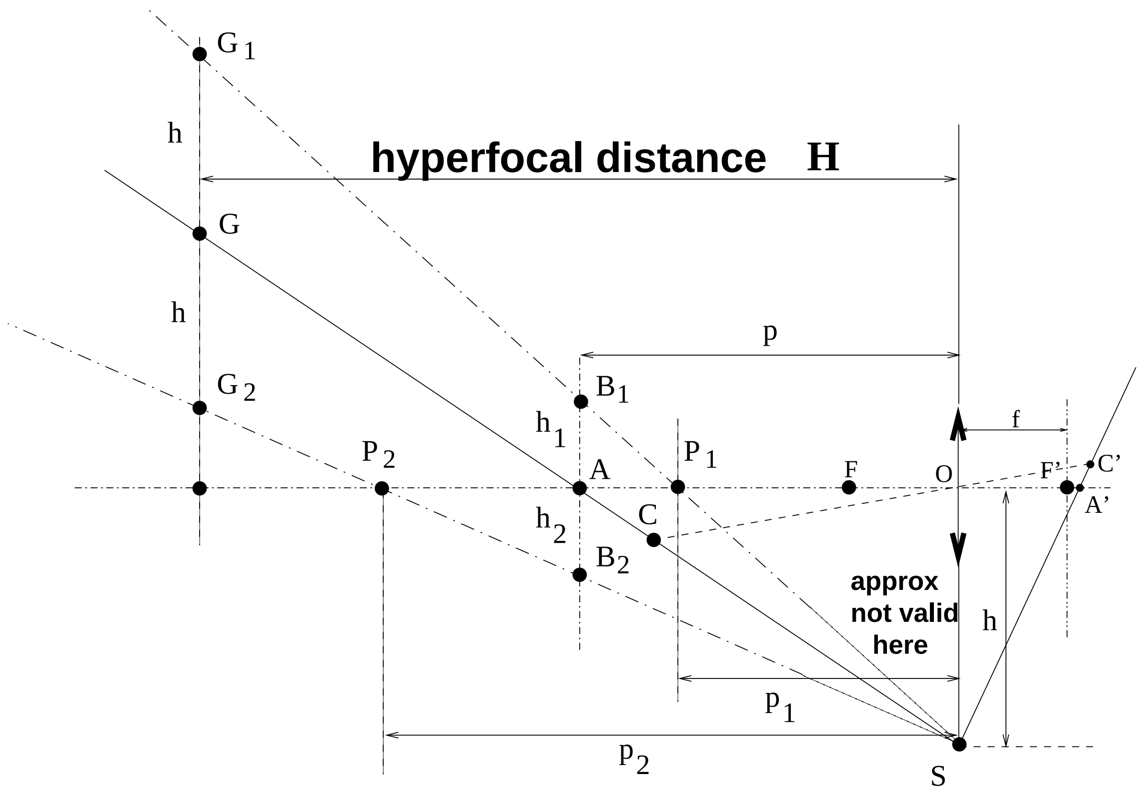


Figure 4: Position of slanted planes of acceptable sharpness, for distant objects, according to this simplest model: fit the original lens with "close-up" lenses of focal length $(\pm H)$

distance H from the lens plane (fig.4). Considering homothetic triangles G_1GS vs. B_1AS and GG_2S vs. AB_2S, we eventually get

$$GG_1 = GG_2 = h \tag{6}$$

a nice result given by Harold M. MERKLINGER in ref.[4]. Note that for far distant objects, the image point A' on the optical axis is located very close to the image focal point F'; thus the distance h, hard to estimate in practice, can be computed from the "camera triangle" OSA' from the focal length f and the estimated tilt angle $\widehat{OSA'}$ as: $\tan(\widehat{OSA'}) \simeq \frac{f}{h}$. For small tilt angles $\tan(\widehat{OSA'}) \simeq \sin(\widehat{OSA'})$, which eventually yields the same result as in reference [4], where the diagram is drawn with a reference line perpendicular to the *film* plane (hence a $sine$ instead of a $tangent$) instead of the lens plane like here.

2 Application to the problem of Depth-of-Focus

Another classical photographic problem is the determination of *Depth-of-Focus*. The question is: *for a given, fixed, object plane, what is the mechanical tolerance on film position in order to get a good image, within certain acceptable tolerances?* The following (figure 5) yields the solution, at least to start with the case of an object plane perpendicular to the optical axis and an image plane parallel to the object plane.

If p' denotes the film position for an ideally sharp image of an object plane at a distance p, the two acceptable limit film plane positions p'_1 and p'_2 are given by

$$p'_1 = p'\left(1 + \frac{f}{H}\right) \; ; \; p'_2 = p'\left(1 - \frac{f}{H}\right) \tag{7}$$

In order to keep the derivation as simple as possible and keep the equivalence with a true optical image formation valid, we need an additional but reasonable approximation, namely that the hyperfocal distance H is much greater that the focal length f. This is what happens in most cases and is argumented in the appendix. Within this approximation, it is found (see details in the appendix), not so surprisingly, that the limit positions p_1 and p_2 as defined above for the Depth-of-Field problem are approximately *the optical conjugates of the positions p'_1 and p'_2* of the Depth-of-Focus problem through the photographic lens of focal length f, as given by DESCARTES formula

$$\frac{1}{p_1} + \frac{1}{p'_1} \simeq \frac{1}{f} \; ; \; \frac{1}{p_2} + \frac{1}{p'_2} \simeq \frac{1}{f} \tag{8}$$

Combining this eq.(8) with the transversal magnification formula p'_1/p or p'_2/p, still the same for pseudo-images (the centre of out-of-focus light spots) as for real images, we find that **in the general case of a slanted object plane, for far distant objects** (so that equation (1) is valid), **the limit positions for the image planes in the Depth-of-Focus problem are given by two slanted**

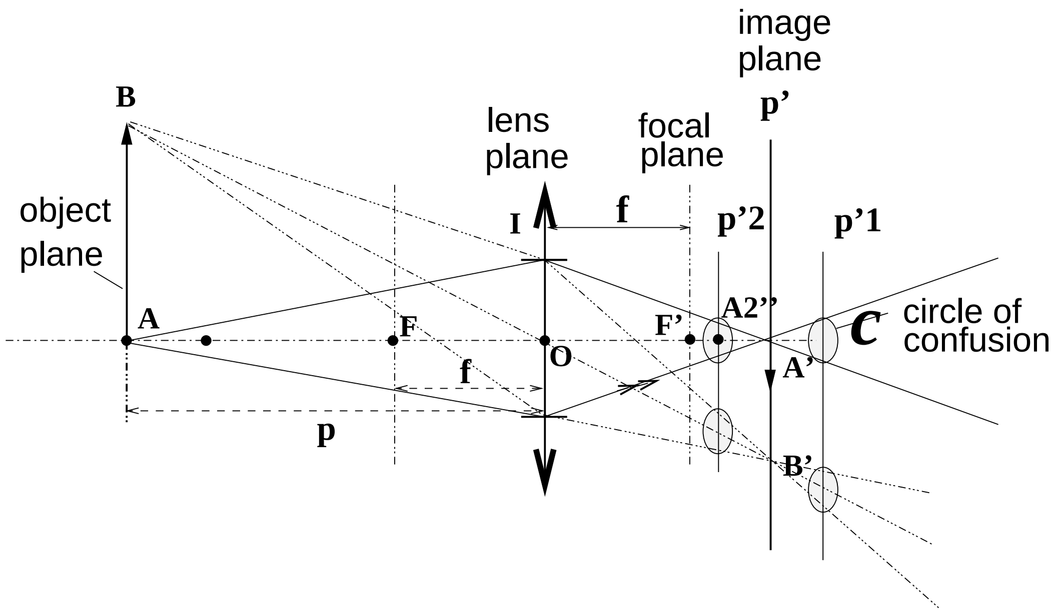


Figure 5: A ray tracing similar to the ones used in the Depth-of-Field problem yields the solution of the Depth-of-Focus problem

planes, those slanted image planes of acceptable sharpness **being the optical conjugates** (through the lens of focal length f) **of the slanted object planes in the Depth-of-Field problem**.

Hence, applying SCHEIMPFLUG'S rule, we conclude again that **those slanted planes intersect together at the same point S** (see fig. 6)

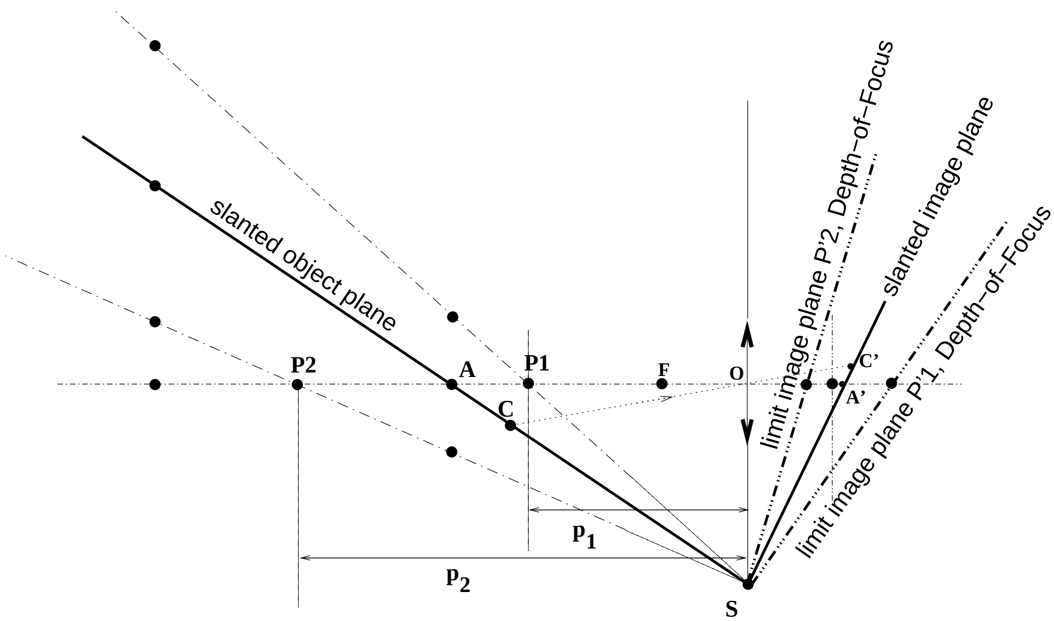
Appendix : depth of field formulae also valid for close-up, reasonable limits for the choice of the circle of confusion c

Depth-of-Field formulae valid for close-up

From NEWTON's object-image formulae $s \times s' = f \times f = f^2$ it is not too difficult (although rather lengthy) for an object AB perpendicular to the optical axis to derive more general formulae giving the position p_1 and p_2 of the planes of acceptable sharpness around a given position of the object p (as measured from the lens plane, see fig.1).

Those exact formulae (9) and (10) are used in a html-javascript [7] and a downloadable spreadsheet [8] on Henri Peyre's French web site. Another derivation, strictly equivalent, is proposed by Nicholas V. Sushkin [6] offering an in-line graph.

There is however a restriction: those formulae will be also valid for a *thick compound* lens where the *pupil planes* are located not too far from the *nodal planes* identical to *principal planes*



P1 and P'1 are optically conjugated through the camera lens of focal length f
same for P2 and P'2, when H >> f, and p >> f

Figure 6: For a slanted object plane AC located far from the lens, and when the hyperfocal distance is much greater than the focal length, the image planes of acceptable sharpness SP_1' and SP_2' in the Depth-of-Focus problem are the optical conjugates of the slanted Depth-of-Field object planes SP_1 and SP_2 through the camera lens

in air. This is the case obviously for a single lens element or a cemented doublet, but also for quasi-symmetric view camera lenses; however for asymmetric lenses or more generally speaking for a lens where pupil planes are far from nodal planes, an extreme case being, for example, so-called *telecentric lenses*, this classical depth-of-field approach is no longer valid. Another ray tracing diagram has to be taken into account; of course depth-of-field will increase when stopping down such a lens, but this will not be *quantitatively* described by equations (9) or (10).

Assuming a given value for c, the diameter of the circle of confusion, a derivation not shown here yields the following (and surprisingly simple) result, which is presented in a slightly different but strictly equivalent form by Nicholas V. Sushkin on his web site [6]

$$\frac{1}{p_1} = \frac{1}{p} + \frac{1}{H}\left(1 - \frac{f}{p}\right) \; ; \; \frac{1}{p_2} = \frac{1}{p} - \frac{1}{H}\left(1 - \frac{f}{p}\right) \tag{9}$$

these formulae can be also written as

$$p_1 = \frac{H.p}{H + (p - f)} \; ; \; p_2 = \frac{H.p}{H - (p - f)} \tag{10}$$

where (see fig.1) f is the focal length of the lens (here considered as a single, positive, thin lens element) p the position (measured from the lens plane O) of the object plane AB, assumed to be perpendicular to the optical axis.

Then p_1 is the position of the *near* limit plane of sharpness and p_2 the position of the *far* limit plane of sharpness. It should be noted in eqs.(9), that all distances p, p_1, and p_2 are (positive) distances measured with respect to the *lens plane plane* O. Here, for a thin positive lens, O is identical to the principal planes. No problem with a thick compound lens if pupillar planes are not too far from principal planes, re-starting from the single thin lens element you just have to "separate" "virtually" the object side from the image side by a distance equal to the (positive of negative) spacing between principal planes.

Definition of the "true" hyperfocal distance

Let us first point out that there is a subtle difference in what appears as the "true" hyperfocal distance when exact formulae are used. If one tries in (9) or (10) to find the proper distance p for which p_2 goes to infinity, the value of $H + f$ is found instead of H in the conventional approach. In this case, the near limit of acceptable sharpness will be $H_{true} = (H + f)/2$. In practice as soon as H is much greater than $5f$, the difference is not meaningful. It could be possible to re-write equations (9) and (10) as a function of H_{true}, but we eventually prefer to denote by *hyperfocal distance* the well-accepted value $H = \dfrac{f.f}{N.c}$ since it naturally comes out of the computation, and as it is referred to in many classical photographic books.

With this assumption on pupillar planes, the formulae given in eq.(9) are derived from NEWTON's formulae within the only, non-restrictive, reasonable approximation that the circle of confusion c (in the range of 20 to 150 microns) is smaller than the diameter of the exit pupil f/N.

Taking $c < 0.5\, f/N$ sounds reasonable. For example with $f = 100$mm, the aperture diameter $a = f/N$ should be smaller than $f/100$ to be smaller than one millimetre, whereas conventional values for c never exceed 0.5mm.

It is also possible to think again about the significance of the hyperfocal distance H by re-introducing the value a of the lens aperture diameter, $a = f/N$. The following expression is obtained: *H/f = a/c*, in other words the ratio between the hyperfocal distance H and the focal length f is equal to the ratio between the lens aperture diameter a and the circle of confusion c. In most practical cases, H is much greater than f. Considering a limit case where H could be close to f, although acceptable from a geometrical point of view, would yield values for c that are too big to be acceptable: for example if c can be as big as $a/2$, equivalent to $H = 2.f$, the resultant image quality will be terrible.

Let's put in some numerical data to support this idea. Consider a standard focal length f equal (by conventional definition of a standard lens) to the diagonal of the image format; assume that the format is square to simplify. The image size will be equal to 0.7f by 0.7f (diagonal size = 1.4 times the horizontal or vertical size of the square). If we assume that $c = a/2 = f/(2.N)$, the number of equivalent image dots will be only $2 \times 0.7 \times N = 1.4 \times N$ both horizontally or vertically. Even at f/90, $N = 90$ this yields a total number of image points smaller than 20,000 (128×128)!!! Even if this "un-sharp" out-of-focus image concerns only a small fraction of the whole image, such a terrible image quality is clearly unacceptable.

Now that we have shown that it is necessary to limit the upper value for c for image quality reasons, this upper limit being somewhat arbitrary, lets us demonstrate that there is also an absolute, unquestionable, minimum value for c due to diffraction effects.

This pure geometrical DOF approach is valid as long as diffraction effects are neglected. Considering a value equal to N microns ($1.22 \times N \times \lambda$, with $\lambda = 0.8\mu m$ in the worst case) for a diffraction spot in the image plane, the other reasonable condition is $c(\text{in microns}) < N\text{microns}$. For example in medium 6x6cm format with $c = 50\mu$m, f/32 is a reasonable f-stop whereas f/64 is irrelevant to the present purely geometrical approach. In 4"x5" format taking $c = 150\mu$m, f/128 will be the smallest non-diffractive aperture for depth-of-field computations.

In macro work at 1:1 ratio (2f-2f), DOF does not depend on the focal length

With all above-mentioned assumptions, equation (9) is valid *even* for short distances p as in macro work, with $p > f$ of course to get a real image. This will be in fact irrelevant to our purpose to find a simple expression and graphical interpretation for far-distant objects, but is of practical use in macro- and micro-photography. For example when $p = 2f$ at 1:1 magnification ratio, the total depth of field is given by $p_2 - p_1 = 4.N.c$, and is totally independent from the focal length, a well-know result.

A numerical computation in agreement with Stroebel's diagrams

Unfortunately there is probably nothing really simple in terms of understanding *geometrically* depth-of-field zones for close-up when the film plane is tilted at a high angle with respect to the optical axis.

From eq.(9) we easily derive a limit form valid for far distant objects, i.e. p much greater than f i.e. $p \gg f$. In this case we can write that $f/p \ll 1$ and $1 - \frac{f}{p} \simeq 1$, $H_{true} = H + f \simeq f$. This yields the well-known expressions of eq.(2).

To go a little further, a numerical computation and graphical computer plot (fig.7) is required. However it is interesting to find the origin of the diagram presented in STROEBEL's excellent reference book [5], stating that limit planes of acceptable sharpness intersect all in the same *pivot point* $\mathbf{P_f}$ located not in the lens plane (like in our approximate model here) but also in the slanted object plane AS, *and one focal length* ahead of the "regular" Scheimpflug's pivot point **S** (figure 7). Without any calculation when p decreases down to the limit value $p = f$, it is easy to see from eq.(9) that both values p_1 and p_2 become equal to f this defining the *pivot point Pf*.

From the computed diagram, here plotted the particular value of $c = f/1000$ ($c = f/1750$ is mentioned sometimes and is more stringent), the simplified approach of the "plus or minus H" close-up lens (yielding slanted planes at large distances p) still holds remarkably well at f/16, even in the macro range. However at f/64 in the close-up range the exact calculation will be required, at least for those inclined to the highest degree of precision, the approximate model being still an excellent starting point to manually refine the focus for slanted SCHEIMPFLUG'S planes. This has been computed with a very simple `gnuplot` [9] freeware script, and will be gladly mailed to all interested readers.

2.1 Depth-of-Focus formulae

Starting from equation (7) defining depth-of-focus limits without approximation, and combining with the exact DOF formulae (9) and (10) yields a complicated expression

$$\frac{1}{p_1} + \frac{1}{p_1'} = \frac{1}{f}\left(1 + \left(1 - \frac{f}{p}\right)\left(\frac{f^2}{H(H+f)}\right)\right) \tag{11}$$

$$\frac{1}{p_2} + \frac{1}{p_2'} = \frac{1}{f}\left(1 + \left(1 - \frac{f}{p}\right)\left(\frac{f^2}{H(H-f)}\right)\right) \tag{12}$$

which would be useless except that its limit form when $H \gg f$ is nothing but equation (8), the additional term inside the bracket vanishing as f^2/H^2. In most photographic situations, with a circle of confusion smaller than $f/1000$, the correcting factor is also of a magnitude smaller than $1/1000$. Equation (11) is actually very close to DESCARTES formula (8) connecting p_1 to p_1' and p_2 to p_2', as long as the basic DOF equation (1) is valid, namely when $p \gg f$ a common photographic situation except in macro work.

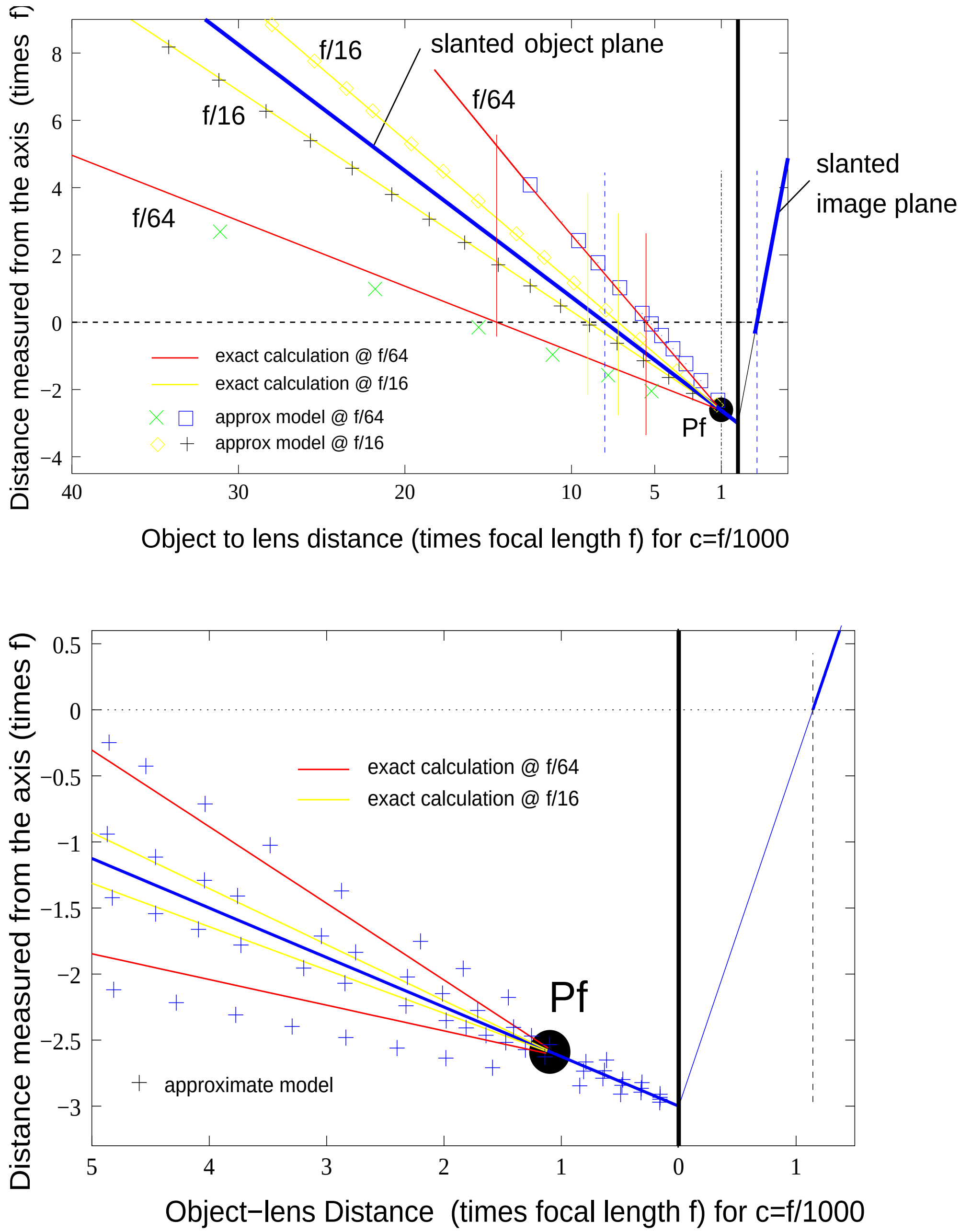


Figure 7: A better determination of slanted object planes of acceptable sharpness, with a pivot point located one focal length ahead of the lens, according to Stroebel (ref.[5]) and re-calculated numerically from eqs. (9)

Equation (7) yields the expression for the *total depth of focus* $p'_1 - p'_2$ equal to $2p'\,f/H$. Subtituing H by its value $f^2/(Nc)$ yields the expression $2Nc\,p'/f$. In the case of the 1:1 magnification ratio (2f-2f), $p' = 2f$, the same value for depth-of-focus or depth-of-field is found i.e. $4Nc$, which makes sense considering the perfect object-image symmetry at 1:1 ratio.

In the practical case of far distant objects, p' will be very close to one focal length f; in this case the total depth-of-focus is found close to $2Nc$. Surprisingly this result does not depend on the choice of focal length f but only the conventional value for c. In other words, for a given film format or a given camera (35mm, medium format, large format) if the same circle of confusion is chosen for all lenses covering a given format with the same camera body, the conclusion, under those assumptions, is that the choice of focal length has no influence on the total depth of focus for far distant objects.

However, in order to peacefully conclude on a potentially controversial subject, the conventional value chosen for c increases somewhat proportionally to the standard focal length when changing from 35 mm to medium and large formats; in a sense it can also be said that depth-of-focus is larger in large format than in small format. How this "large format advantage" actually helps getting better images in large format for given mechanical manufacturing tolerances or film flatness cannot be simply inferred without deeper investigations.

Acknowledgements

I am very grateful to Yves Colombe for his explanations about subtle pupil effects in a non-symmetric or telecentric lens. In the more general case, the projection of the exit pupil actually defines out-of-focus "pseudo-images" of object points. In the general case, those "pseudo-images" do not obey the classical depth-of-field formulae nor, of course, basic object-to-true-image relations. Simon Clément has pointed to me the fact the the "true" hyperfocal distance becomes $H + f$ when the exact DOF formulae are in use. The subtle question of the transversal magnification for out-of-focus pseudo-images has been clarified by a passionate debate on one of the US Internet discussion groups on photography, the key point being mentioned by Andrey VOROBYOV [10].

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Abstract. By using geometry, a fairly complete analysis of Kemeny's rule (KR) is obtained. It is shown that the Borda Count (BC) always ranks the KR winner above the KR loser, and, conversely, KR always ranks the BC winner above the BC loser. Such KR relationships fail to hold for other positional methods. The geometric reasons why KR enjoys remarkably consistent election rankings as candidates are added or dropped are explained. The power of this KR consistency is demonstrated by comparing KRâ€¦ CONTINUE READING. The Scheimpflug camera offers a wide range of applications in the field of typical close-range photogrammetry, particle image velocity, and digital image correlation due to the fact that the depth-of-view of Scheimpflug camera can be greatly extended according to the Scheimpflug condition. Yet, the conventional calibration methods are not applicable in this case because the assumptions used by classical calibration methodologies are not valid anymore for cameras undergoing Scheimpflug condition. Therefore, various methods have been investigated to solve the problem over the last few years. How This Field Guide derives from the treatment of geometrical optics that has evolved from both the undergraduate and graduate programs at the Optical Sciences Center at the University of Arizona. The development is both rigorous and complete, and it features a consistent notation and sign convention. This volume covers Gaussian imagery, paraxial optics, first-order optical system design, system examples, illumination, chromatic effects, and an introduction to aberrations.