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Review of *Iterative Solution Methods* by O. Axelsson*

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As stated in the Preface, this book is devoted to the numerical solution of linear algebraic systems. Although it deals mainly with iterative methods, some results concerning direct solvers are also presented. The fundamental role of preconditioning is stressed throughout the text. The so-called generalized conjugate-gradient methods (a class of methods proposed and developed by the author) and some preconditioning techniques are exposed in a very detailed way. The book also covers parts of matrix theory and the theory of eigenvalue computation related to the main topic.

The aim of the author was to address both the algorithms and their theory. In my opinion, the theoretical point of view dominates. A deep understanding of the theoretical background of the methods, together with the development of general concepts which would allow one to describe in a uniform way large families of methods, is stressed, rather than discussion of specific implementation details.

The book is organized in 13 chapters and 3 appendices. Each chapter begins with an introduction giving the essence of the material exposed later in the text. For most of the chapters, the introduction represents a comprehensive and well-written summary of fundamental ideas and results in the given area. Each chapter ends with its own list of references. The first seven chapters and Appendix A have been used by the author as a textbook for an introductory course in numerical linear algebra, but, as he pointed out, it requires students who are not afraid of theory. These chapters contain a large number of exercises, most of which systematically extend the covered material. The second part of the book presents some recent results in the iterative solution of linear systems. It is devoted to the preconditioned conjugate-gradient-type methods, which reflects long-term interests and research activities of the author.

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Chapter 1 briefly reviews basic concepts of direct methods and recalls some general results concerning the problem of solving linear algebraic systems. Chapters 2, 3, and 4 summarize the background on matrix eigenvalues, special classes of matrices, and Schur complements which is used in the later parts of the book. At this point I must go forward to Appendix A. It contains the basic information about matrix norms, errors in solving linear systems, and computation of eigenvalues. I do not understand why the material of Appendix A (or at least part of it) was not included in the introductory part of the book. That would allow the author, e.g., to define and use the condition number of a matrix in a natural and obvious way (using the special definition relevant to symmetric positive definite matrices throughout the book, while large parts of the text deal with matrices which are not symmetric, seems a bit confusing to me). Presenting the elementary properties of determinants would also improve the readability of the book for students.

Chapters 5 and 6 are devoted to classical iterative methods and their rate of convergence, including the convergence theory for the SOR method for symmetric and positive definite matrices. The material is explained in a nice way.

Chapters 7–10 present and analyze several preconditioning techniques. Chapter 7 considers the incomplete factorization and block incomplete factorization methods, Chapter 8 approximate matrix inverses and the combination of implicit and explicit preconditionings, and Chapter 9 block diagonal and Schur-complement preconditionings. In Chapter 10, estimates of eigenvalues and condition numbers of preconditioned matrices are analyzed.

The rest of the book is devoted to conjugate-gradient-type methods. Chapter 11 describes the classical results on the conjugate-gradient and Lanczos methods, together with some of their extensions for solving nonsymmetric systems. Chapter 12 presents theory of the generalized conjugate-gradient methods developed by the author. Finally, Chapter 13 summarizes his original results (with several exceptions) describing the rate of convergence of the generalized conjugate-gradient methods. Appendices B and C give some basic facts about Chebyshev polynomials and several inequalities for function of matrices.

This book contains an enormous amount of material. It cannot completely cover, however, the large area of iterative methods. The topics selected by the author are described in detail. Many original approaches and proofs are presented in the text, especially in the parts related to the pioneering work of the author on preconditioning, generalized conjugate-gradient methods, and the characterization of their convergence. Though I will probably not use the first part of the book as a textbook (for the basic course in numerical linear algebra I prefer *Fundamentals of Matrix Computations* by David Watkins), I will certainly use it for preparing lectures on some selected topics (as, e.g., classical iterative methods), and I will profit from exercises collected there. Many historical remarks are valuable for everyone working in the area.

On the other hand, my feeling is that this book does not offer enough information to serve as a firm basis for choosing and efficiently implementing the best method for solving a given specific problem. While some particular methods are described very thoroughly, the other important methods (as, e.g., QMR and GMRES) are hardly mentioned, and no comparison is offered. In my opinion, such comparison would be highly desirable to help the potential user in eliminating the inefficient or unstable algorithms and choosing the proper ones. Moreover, the description of several algorithms is not precise. For example, the GMRES method can be implemented in a more efficient and stable way than the one mentioned on pp. 541–542. In particular, one does not need to compute the approximate solution x^{k+1} in order to check if the norm of the corresponding residual is less then some chosen accuracy level, and the modified Gram-Schmidt method should be at least mentioned in this context.

I have found several other inaccuracies. The following one seems to me serious. It is well known that if the system matrix is normal, then the convergence rate of the conjugate-gradient-type methods is essentially determined by the distribution of its eigenvalues. In the nonnormal case the situation is, however, much less clear. While in Chapter 13 this problem is explicitly indicated and the normal case is distinguished from the nonnormal one, in the previous chapters there are many formulations in which the convergence of (preconditioned) conjugate-gradient-type methods is automatically related to the distribution of eigenvalues without mentioning any assumption or warning about the restricted applicability of the result.

Despite some critical comments, I consider this book a very valuable monograph. I certainly recommend buying it.

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Buy Iterative Solution Methods on Amazon.com âœ“ FREE SHIPPING on qualified orders.Â Review. "...the book is the most complete and interesting study of iterative methods for systems of linear equations to date. I strongly recommend it to any researcher in the field or in any other area in which the solution of large systems of linear equations plays an important role...it will become a standard reference in numerical linear algebra..." Joaquim J. Judice, Mathematical Reviews.Â Axelsson discusses iterative methods that he claims can converge rapidly. While this may not be generally true, the pragmatic researcher might keep Axelsson's ideas in mind, and consider applying them to her problems. The first section of the book is somewhat mundane.